

Determination of $\alpha_s(m_Z)$: N³LL Analysis of Thrust Distribution with Power Corrections

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R.A., M. Fickinger, A. Hoang, V. Mateu, I. Stewart arXiv:1006.3080
R.A., A. Hoang, V. Mateu, M. Schwartz, I. Stewart work in progress



Massachusetts
Institute of
Technology

Outline

Motivation

- Event Shapes analyses in the world average for $\alpha_s(m_Z)$ do not include power corrections with controlled approximations, nor are able to describe the thrust distribution in its whole range
- There is a huge amount of experimental data for thrust, not fully exploited

Thrust analysis

- Factorization theorem for thrust
 - N³LL resummation
 - coherent description of the whole range of thrust distribution
 - Field theoretical treatment of power corrections
- Global tail fit for $\alpha_s(m_Z)$ and first power correction

Comparison with Heavy Jet Mass

- Preliminary results on the analysis of the heavy jet mass distribution

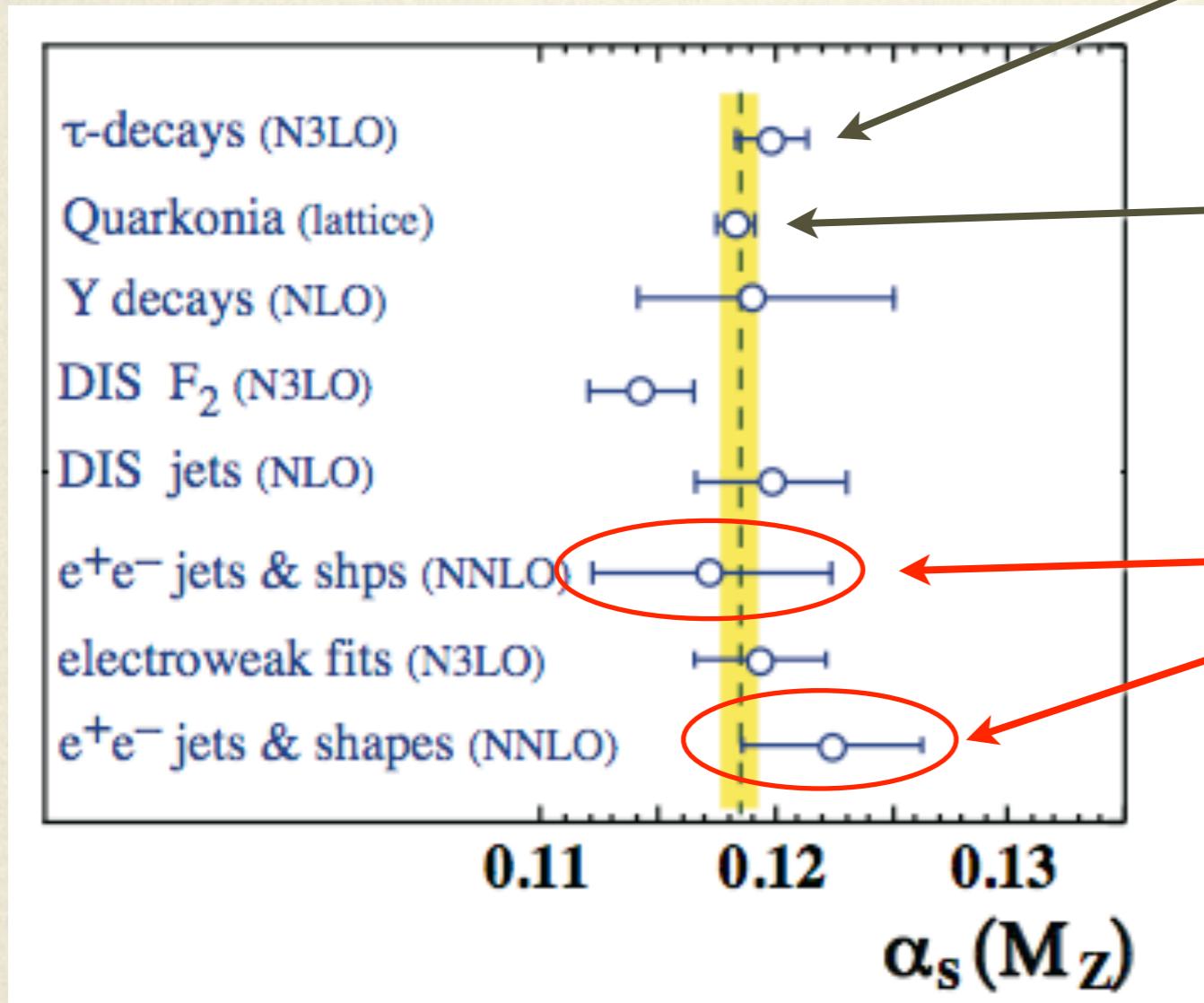
Motivations

Latest World Average

S. Bethke, arXiv:0908.1135
adopted by PDG for 2010

$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

errors inflated to account for variation in literature



fit to Υ -splittings, Wilson loops
 $\alpha_s(m_Z) = 0.1183 \pm 0.0008$
HPQCD 0807.1687

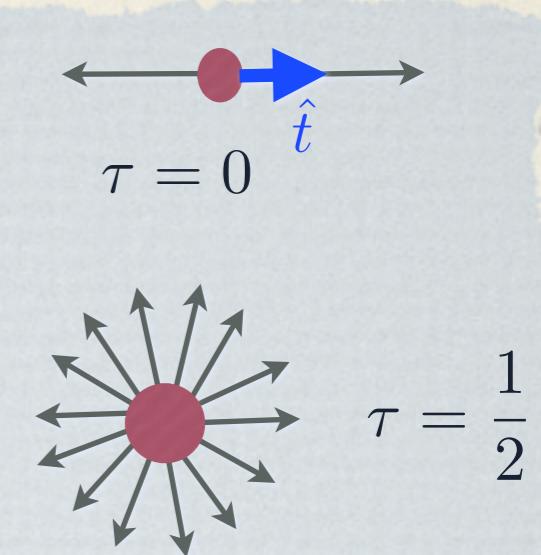
event shape results at $\mathcal{O}(\alpha_s^3)$
(Doesn't include N³LL
resummation nor rigorous
treatment of power
correction)

$$e^+ e^- \xrightarrow{Q} \text{jets}$$

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{Q} \quad \tau = 1 - T$$

Experiment Q values (GeV)

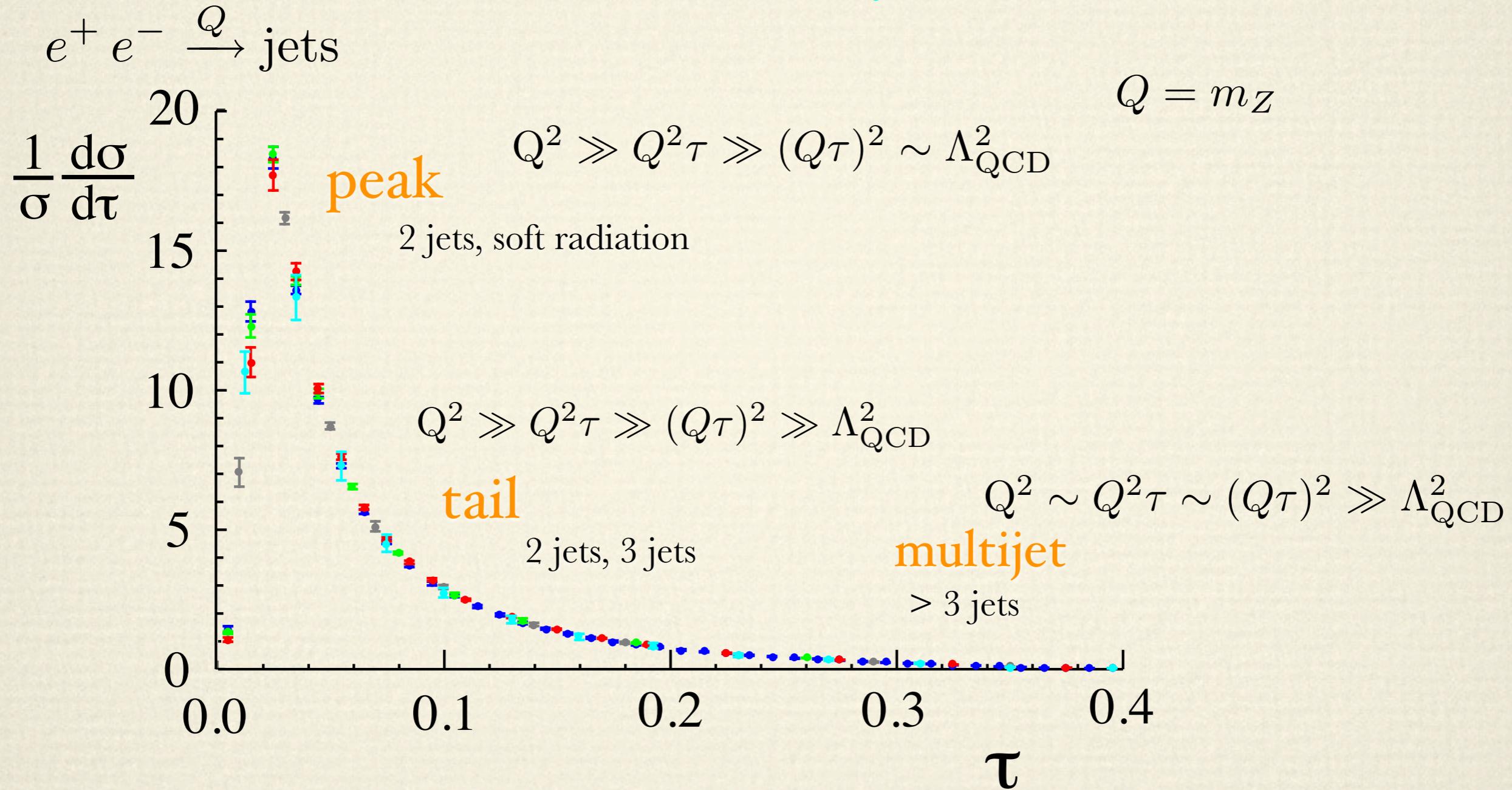
Thrust



	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
LEP	DELPHI	{45.0, 66.0, 76.0, 91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
	OPAL	{91.0, 133.0, 161.0, 172.0, 177.0, 183.0, 189.0, 197.0}
	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLAC	SLD	{91.2}
	TASSO	{14.0, 22.0, 35.0, 44.0}
DESY	JADE	{35.0, 44.0}
KEK	AMY	{55.2}

Thrust

ALEPH, DELPHI, L₃, OPAL, SLD



Thrust Analysis

Factorization Theorem for Thrust

$e^+ e^- \xrightarrow{Q} \text{jets}$

AFHMS (arXiv:1006.3080)

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\text{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\text{QCD}}}{Q}\right) \right]$$

Factorization Theorem for Thrust

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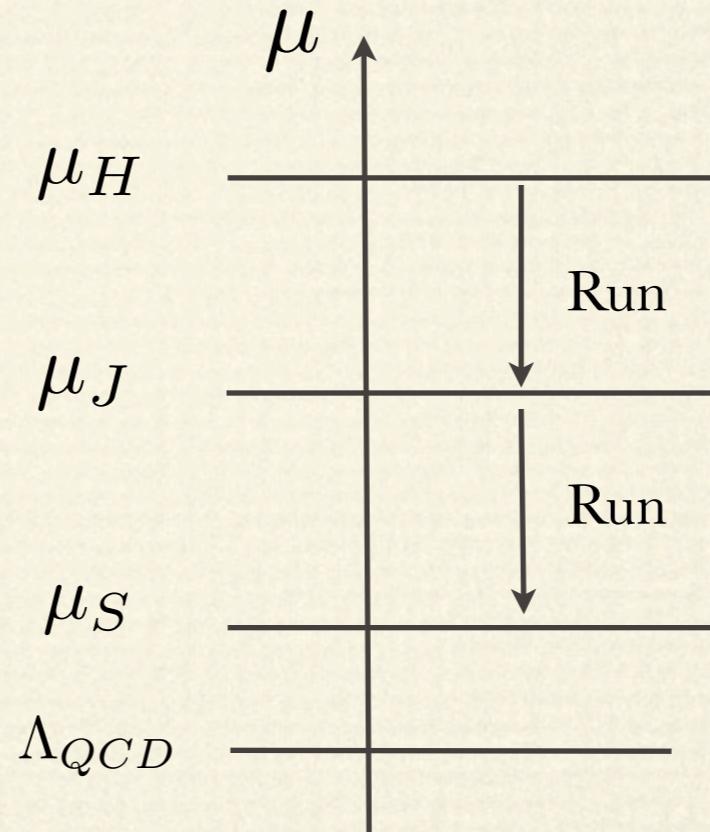
$$\begin{aligned} \frac{d\hat{\sigma}_s}{d\tau} &= \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\log^l \tau}{\tau} \right]_+ \\ &= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S) \end{aligned}$$

singular partonic cross for massless quarks, QCD+QED final states

resummation for singular partonic

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \dots$$

Becher & Schwartz '08 $y = FT(\tau)$



Match QCD to SCET

Integrate out Jet modes

Soft Function OPE

Factorization Theorem for Thrust

$e^+ e^- \xrightarrow{Q} \text{jets}$

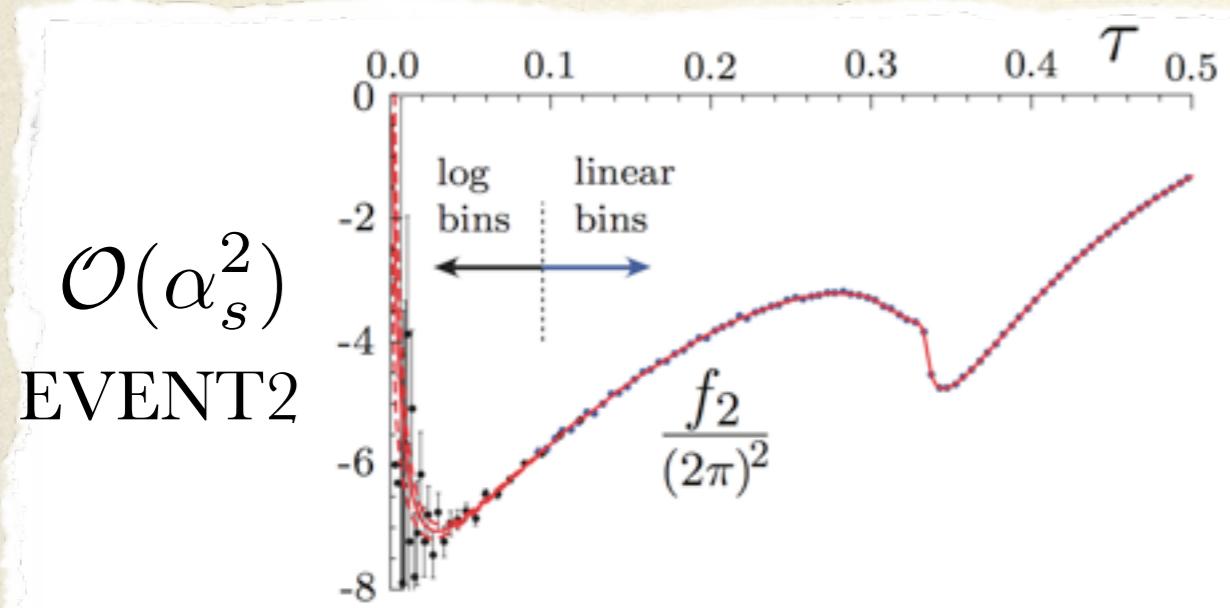
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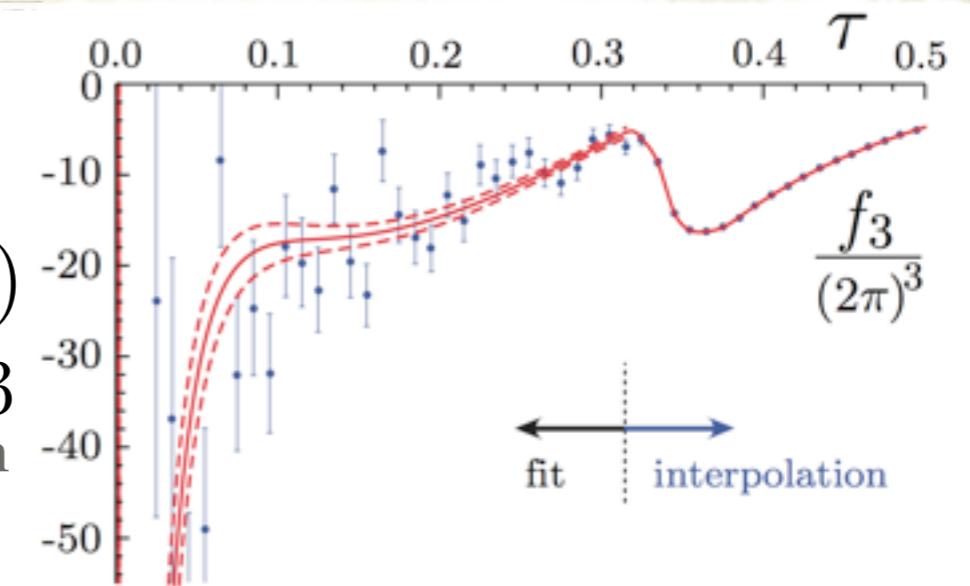
$$\frac{d\hat{\sigma}}{d\tau} \Big|_{\text{fixed order}} = \frac{d\hat{\sigma}}{d\tau} \Big|_{\text{SCET expanded}} + \frac{d\hat{\sigma}_{ns}}{d\tau}$$

$$\frac{d\hat{\sigma}_{ns}}{d\tau} \left(\tau, \frac{\mu_{ns}}{Q} \right) = \sigma_0^I e^{2\frac{\delta(R)}{Q} \frac{\partial}{\partial \tau}} f^I \left(\tau, \frac{\mu_{ns}}{Q} \right)$$

$$f^I(\tau, 1) = \sum_n \frac{\alpha_S^n}{(2\pi)^n} f_n^I(\tau)$$



$\mathcal{O}(\alpha_s^3)$
EERAD3
Gehrmann
et al.



Factorization Theorem for Thrust

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AFHMS (arXiv:1006.3080)

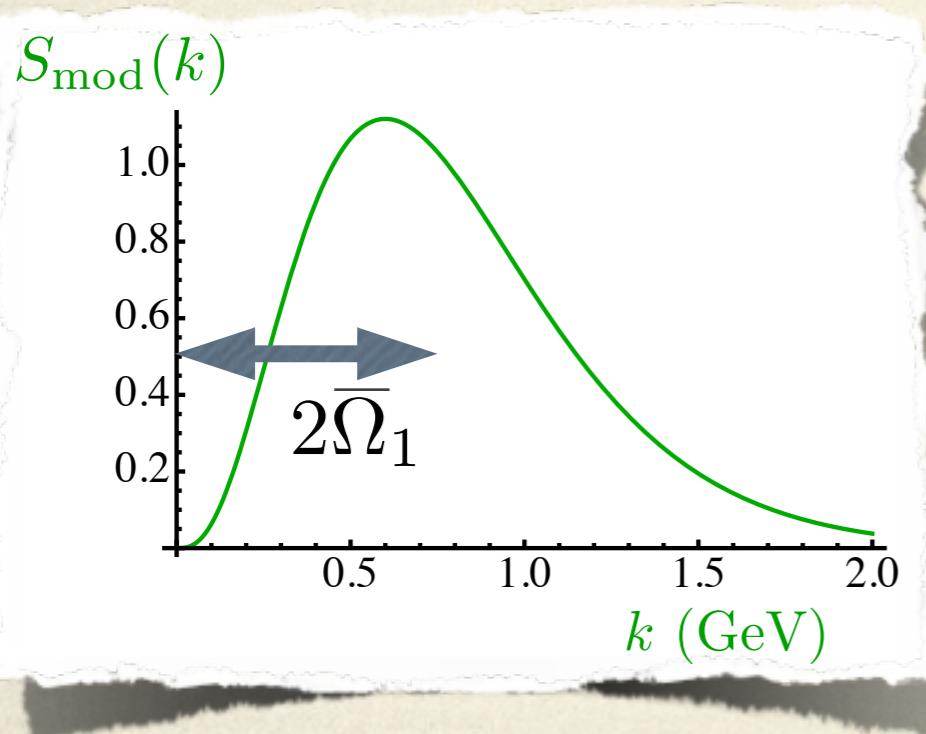
$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\text{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\text{QCD}}}{Q}\right) \right]$$

Non Perturbative Effects

In the tail region, where $\ell_{\text{soft}} \sim Q\tau \gg \Lambda_{QCD}$
the soft function can be expanded as

$$\begin{aligned} S_\tau(Q\tau, \mu) &= \int dk' S_{\text{part}}(Q\tau - k', \mu) S_\tau^{\text{mod}}(k') \\ &= S_{\text{part}}(Q\tau, \mu) - S'_{\text{part}}(Q\tau, \mu) 2\bar{\Omega}_1 + \dots \\ &= S_{\text{part}}(Q\tau - 2\bar{\Omega}_1, \mu) + \dots \end{aligned}$$

the distribution shifts!



$$\begin{aligned} \bar{\Omega}_1 &\equiv \int dk' \left(\frac{k'}{2}\right) S_\tau^{\text{mod}}(k') && \text{MS} \\ &\equiv \frac{1}{2N_c} \langle 0 | \text{tr } \bar{Y}_{\bar{n}}(0) Y_n(0) i\partial_\tau Y_n^\dagger(0) \bar{Y}_{\bar{n}}^\dagger(0) | 0 \rangle \\ &\sim \Lambda_{QCD} \end{aligned}$$

There is still an issue to address

Renormalon subtraction

The partonic soft function S_{part} and the \overline{MS} definition of $\overline{\Omega}_1$ have an $\mathcal{O}(\Lambda_{QCD})$ renormalon ambiguity

Perturbative instability and large order dependence

Renormalon subtraction

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Perturbative instability and large order dependence

Solved introducing a gap parameter Δ :

$$S_\tau^{\text{mod}}(k) \rightarrow S_\tau^{\text{mod}}(k - 2\Delta)$$

and writing

$$\Delta = \bar{\Delta}(R, \mu_S) + \delta(R, \mu_S)$$

Hoang, Stewart

Renormalon Free

$$\delta(R, \mu) = \frac{R}{2} e^{\gamma_E} \frac{d}{d \ln(ix)} [\ln S_\tau(x, \mu)]_{x=(iRe^{\gamma_E})^{-1}}$$

$$= e^{\gamma_E} R \left[-0.849 \frac{\alpha_s(\mu_S)}{4\pi} \log \frac{\mu_S}{R} + \dots \right]$$

Renormalon subtraction

The partonic soft function S_{part} and the \overline{MS} definition of $\bar{\Omega}_1$ have an $\mathcal{O}(\Lambda_{QCD})$ renormalon ambiguity

Perturbative instability and large order dependence

Solved introducing a gap parameter Δ :

$$S_\tau^{\text{mod}}(k) \rightarrow S_\tau^{\text{mod}}(k - 2\Delta)$$

and writing

$$\Delta = \bar{\Delta}(R, \mu_S) + \delta(R, \mu_S) \quad \text{Hoang, Stewart}$$

so,

$$S_\tau(k, \mu) = \int dk' \left[e^{-2\delta \frac{\partial}{\partial k}} S_{\text{part}}(k - k', \mu) \right] S_\tau^{\text{mod}}(k' - 2\bar{\Delta})$$

$\delta(R, \mu_S)$ induces a subtraction series in the cross section and relates the \overline{MS} and the renormalon free definition of Ω_1

$$\Omega_1(R, \mu_S) = \bar{\Omega}_1 - \delta(R, \mu_S)$$

Renormalon Free

In order to avoid large logs, we need $R \sim \mu_S$

Factorization Theorem for Thrust

$e^+ e^- \xrightarrow{Q} \text{jets}$

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$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\text{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\text{QCD}}}{Q}\right) \right]$$

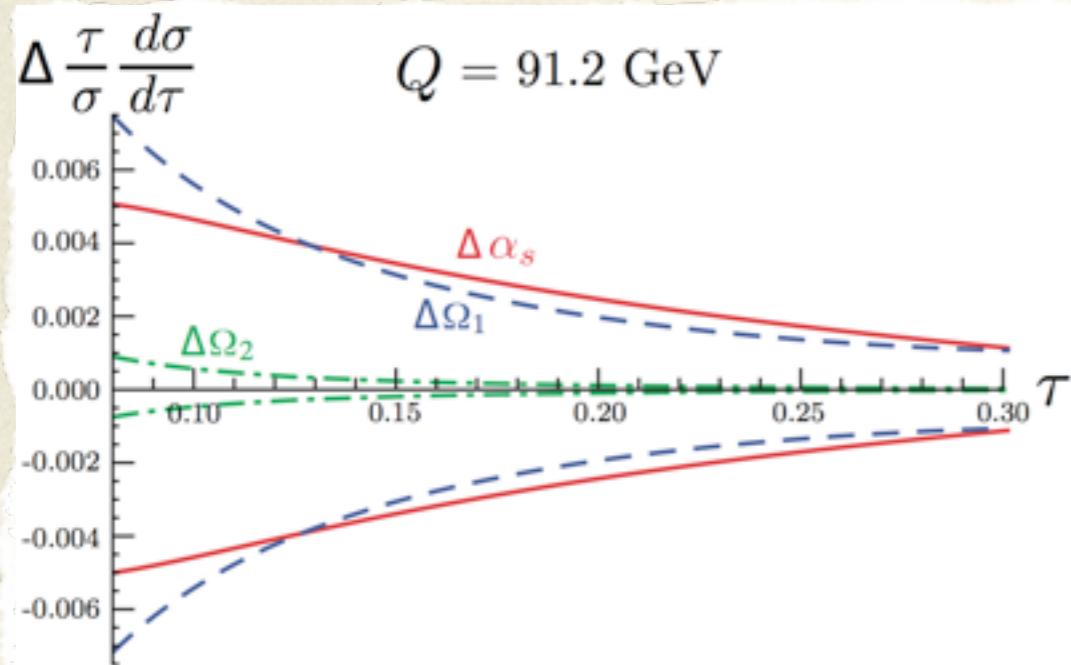
- $\mathcal{O}(\alpha_s^3)$ fixed order (non singular). Event2 $\mathcal{O}(\alpha_s^2)$ and EERAD3 $\mathcal{O}(\alpha_s^3)$
- $\mathcal{O}(\alpha_s^3)$ matrix elements. Axial singlet anomaly. Full hard function at 3 loops
- Resummation at N^3LL . Effective field theory (SCET)

- Field theory matrix elements for renormalon-free power corrections
- Correct theory in peak, tail and multijet regions (profile functions)

- QED effects in Sudakov and FSR at NNLL+ $\mathcal{O}(\alpha_s^2)$ with $\alpha \sim \alpha_s^2$
- Bottom mass corrections with factorization theorem
- Computation of bin cumulants in a meaningful way

Why a global fit? (Many Q values)

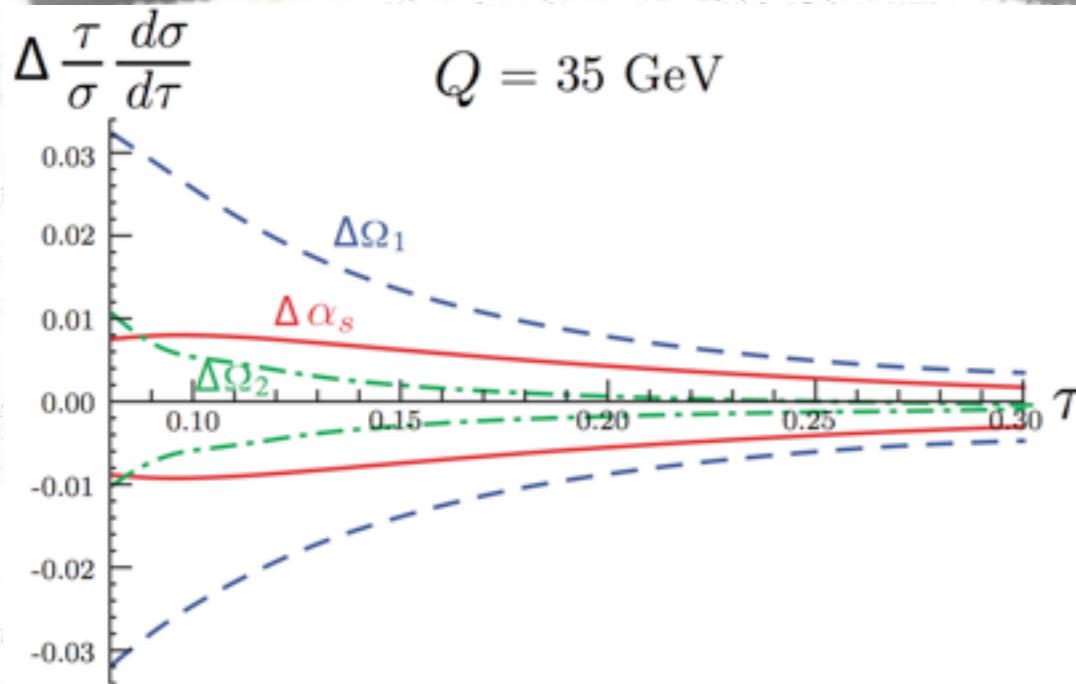
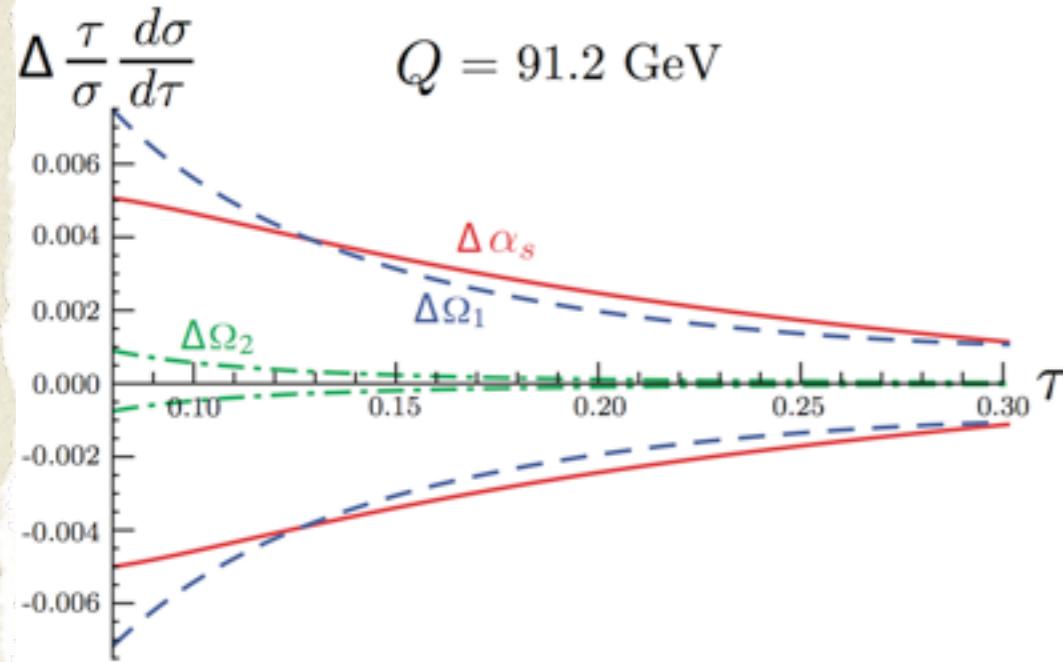
We fit for $\alpha_s(m_Z)$ and Ω_1 simultaneously.



At a single Q , a variation in $\alpha_s(m_Z)$ can always be compensated by a variation in Ω_1 , the two parameters are strongly degenerate

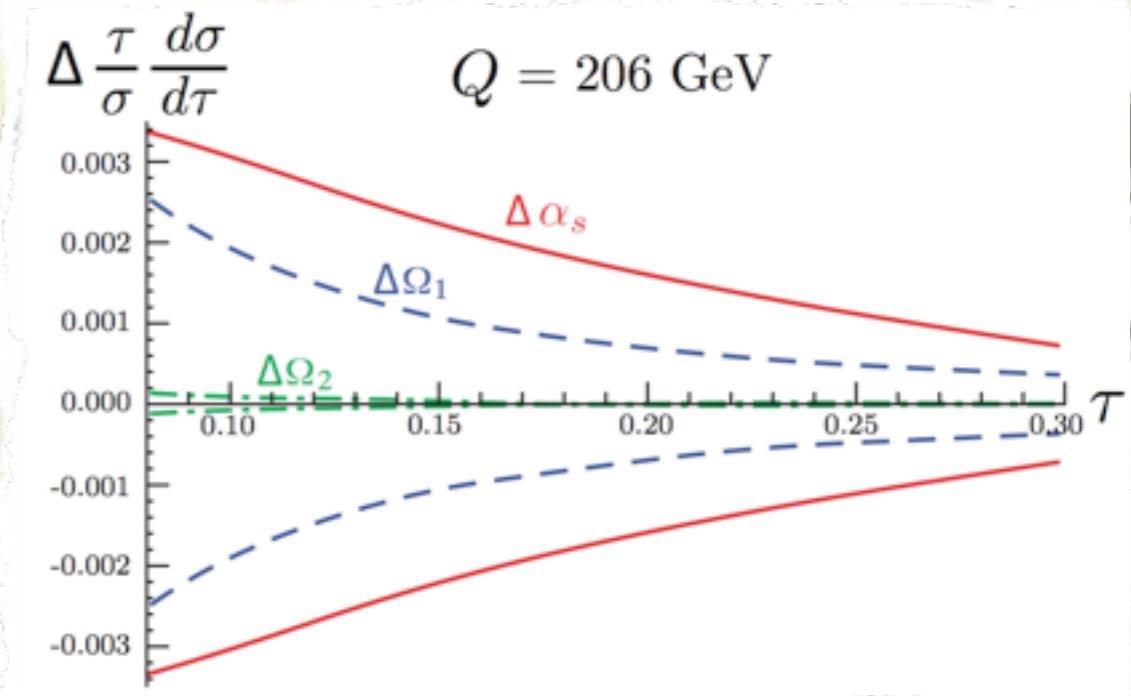
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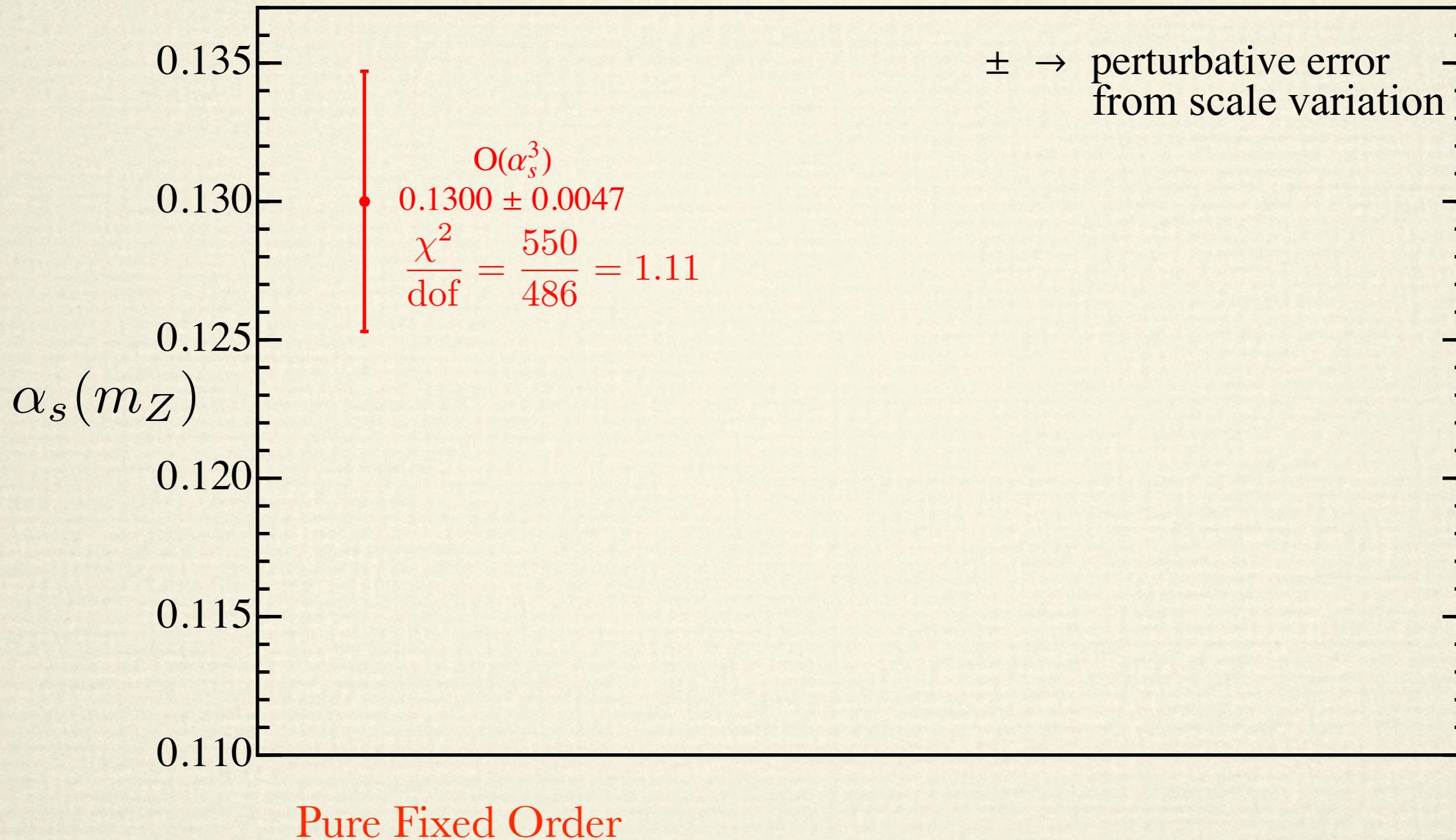
Fitting multiple Q , we can break the degeneracy!

Power correction needed at 20% accuracy to get $\alpha_s(m_Z)$ at the 1% level



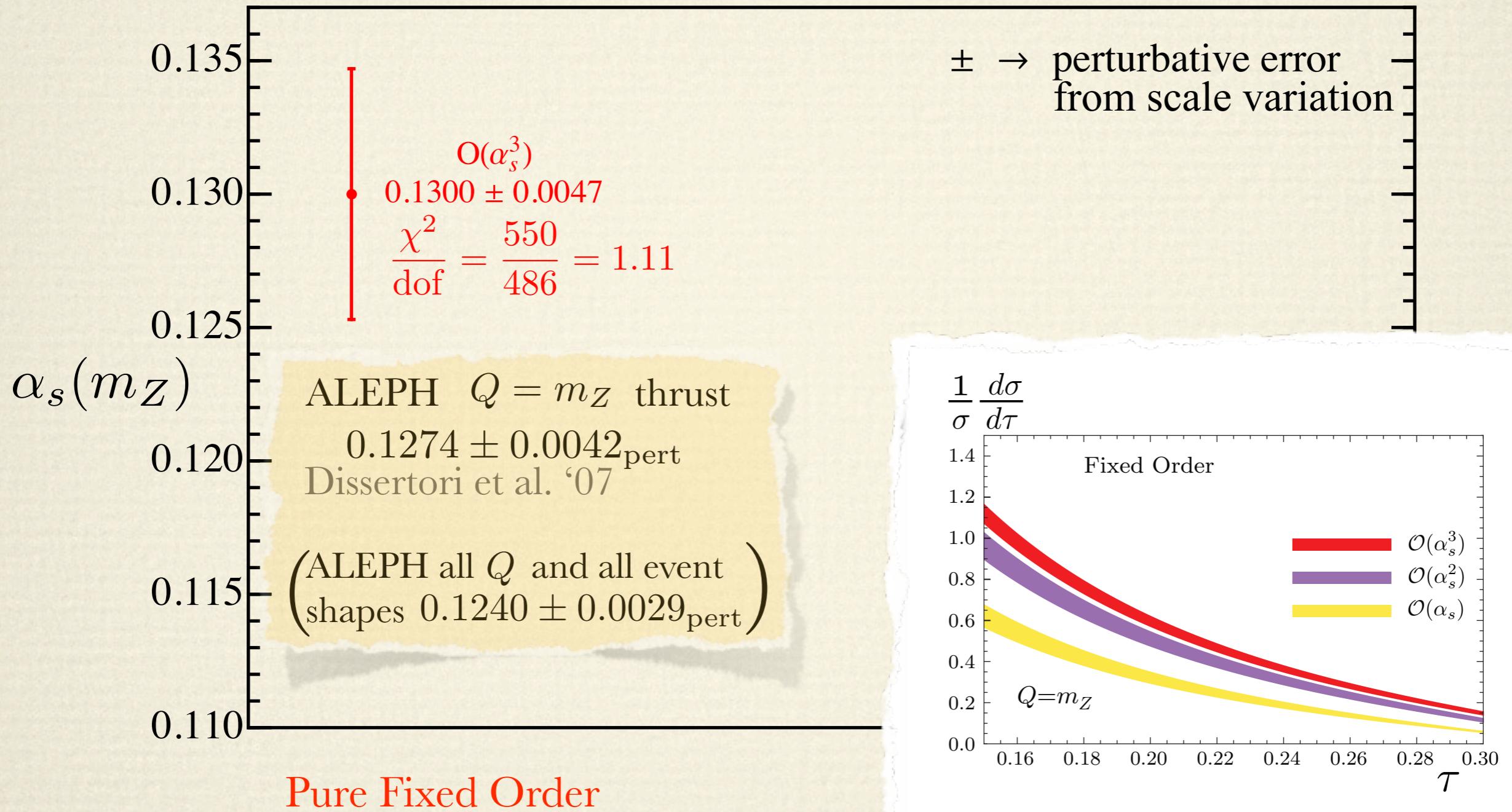
Global tail fit for $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ from global thrust fits



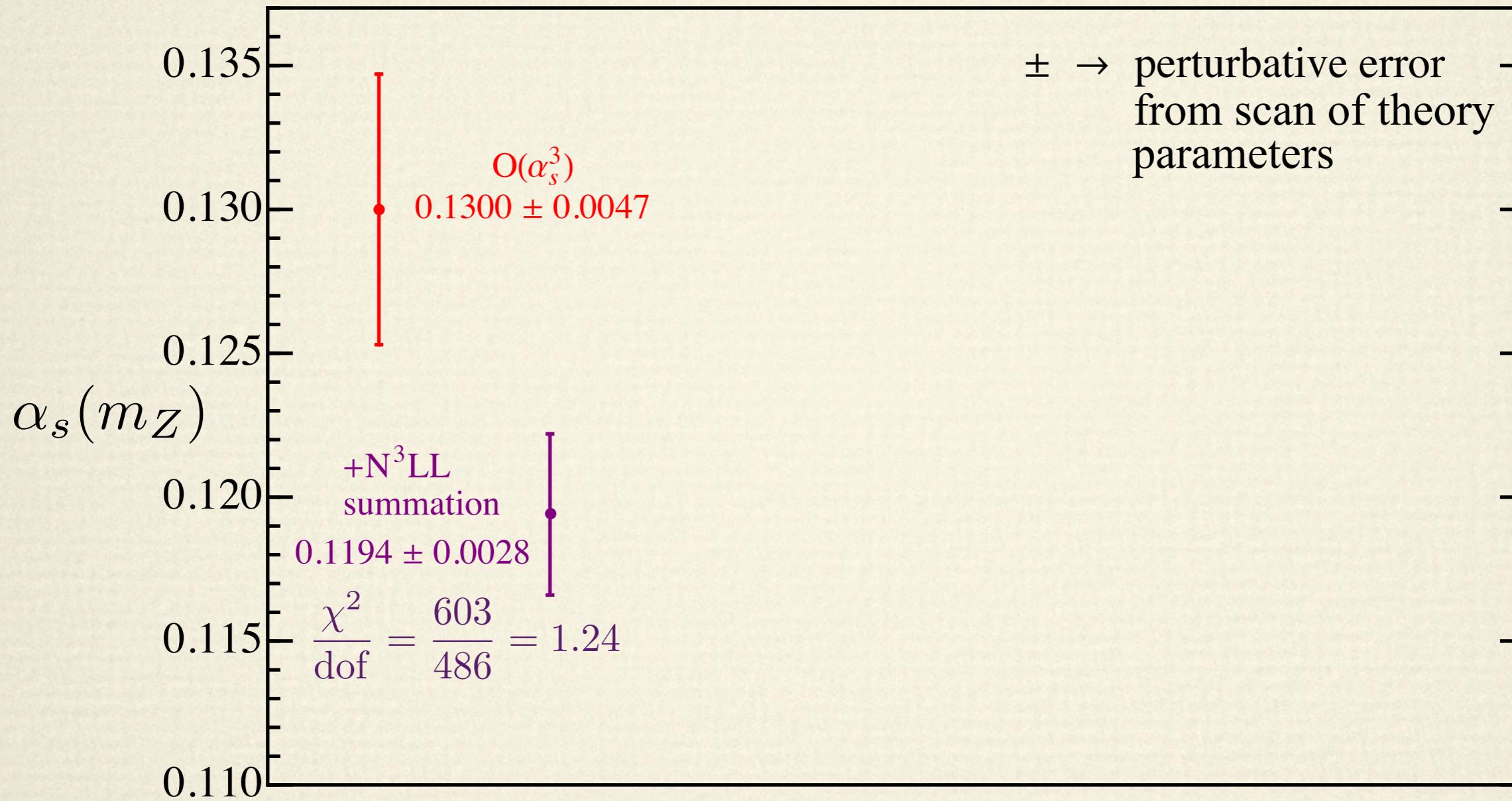
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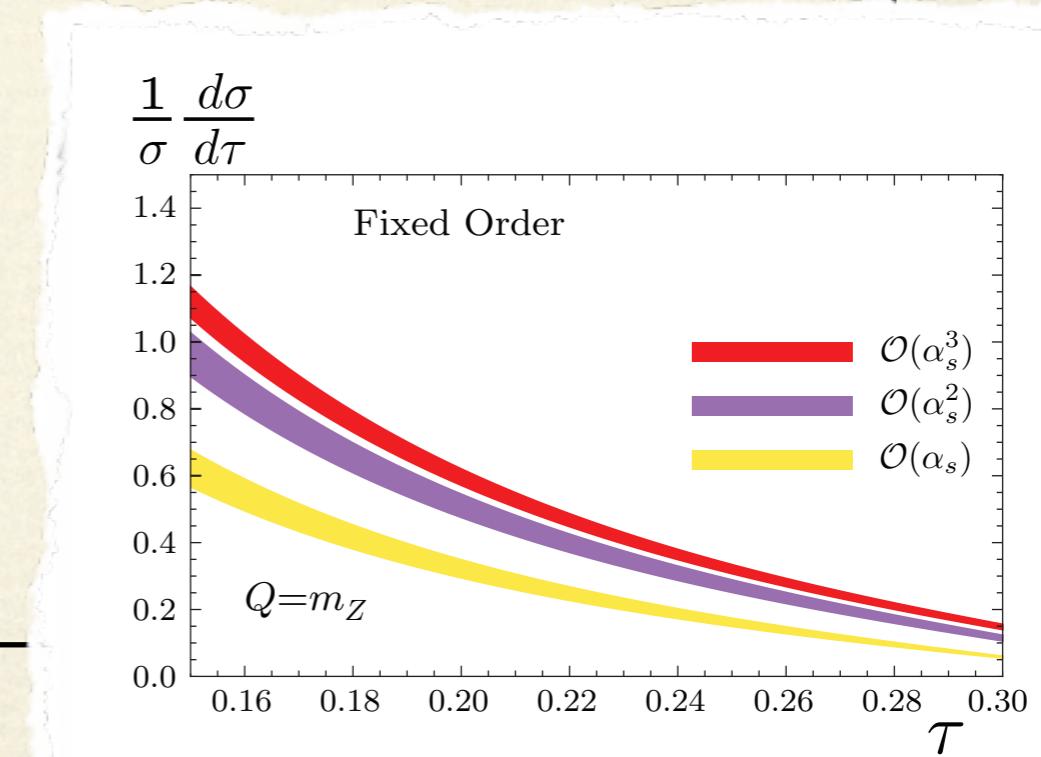
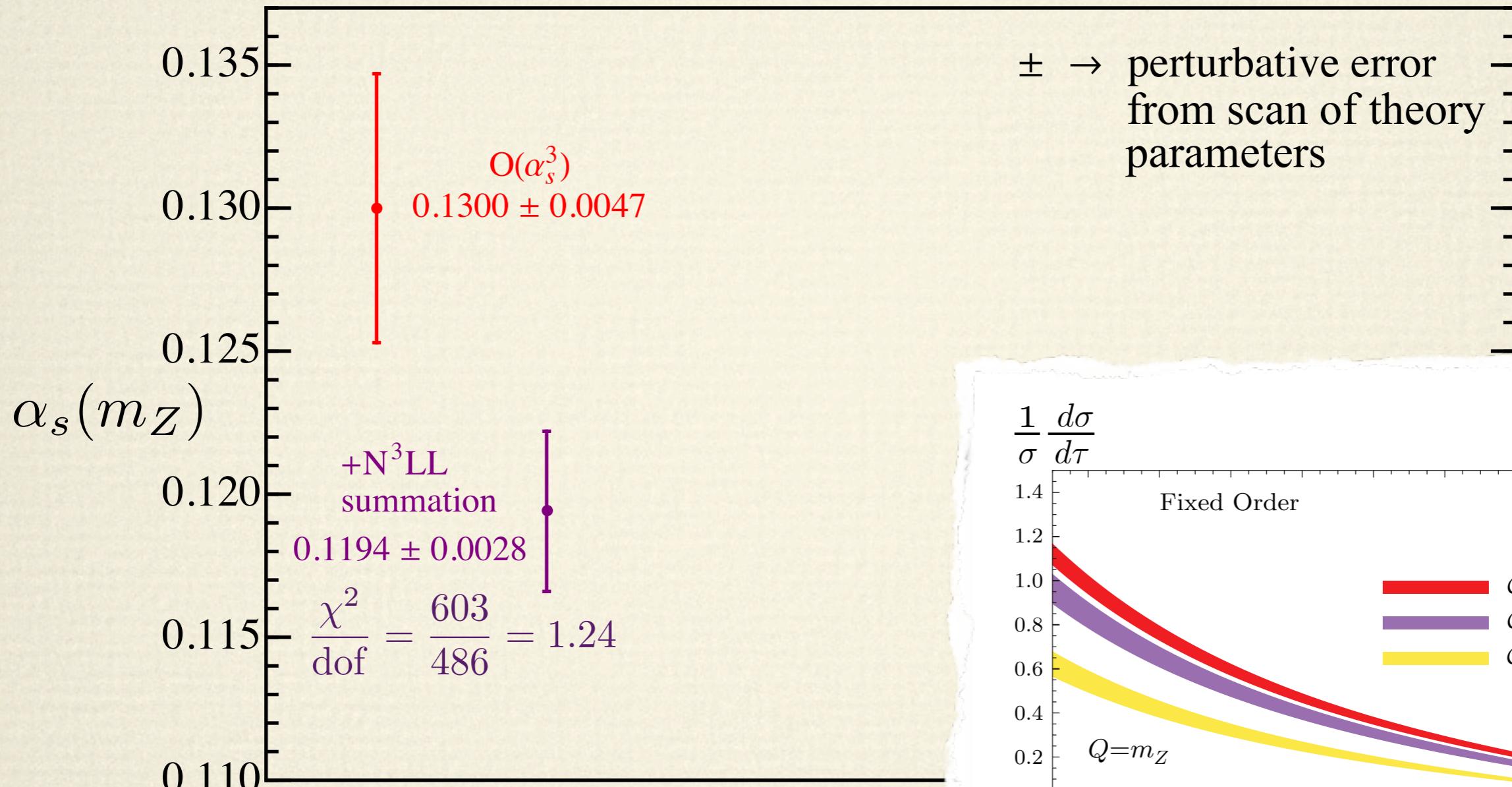
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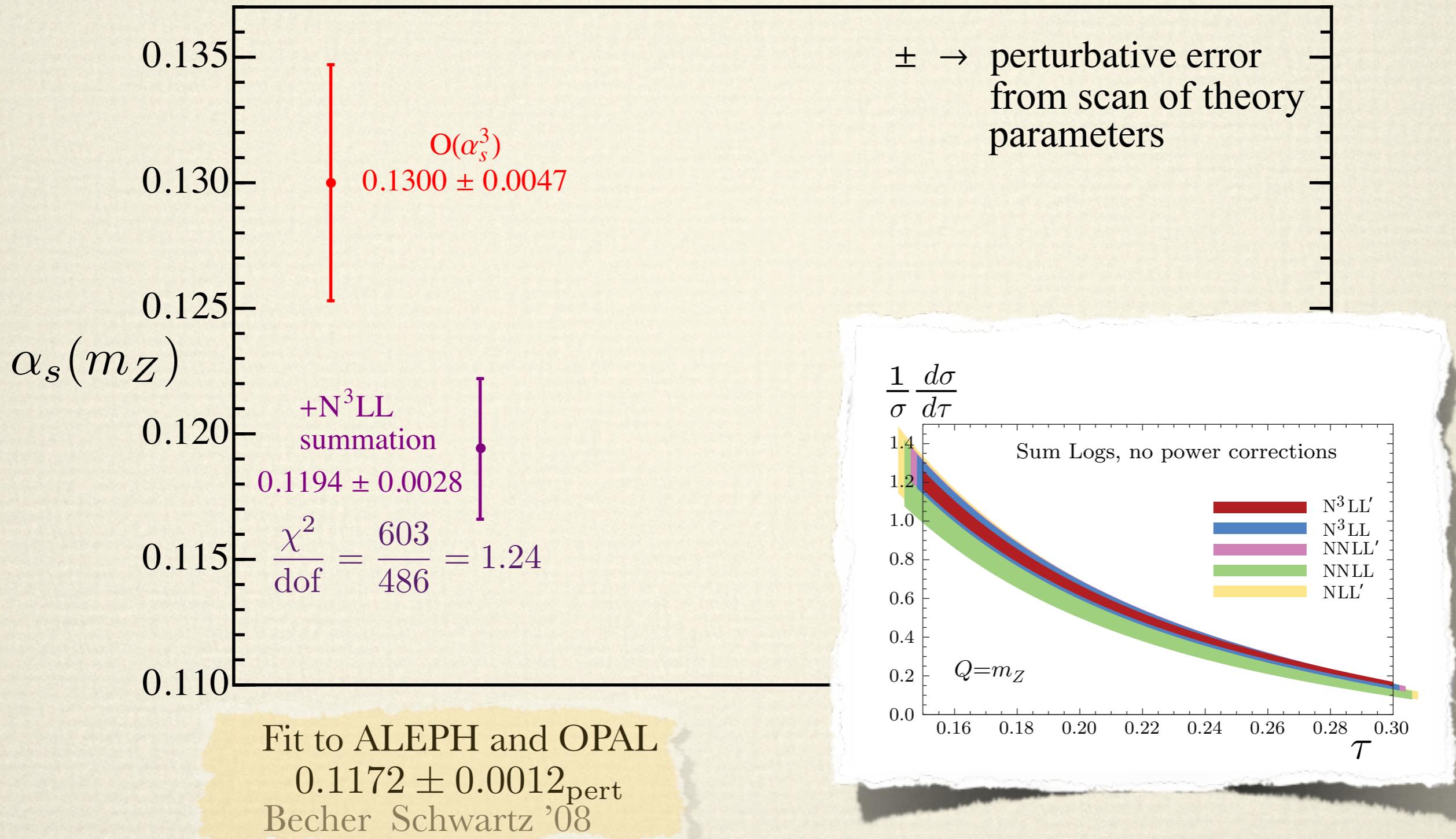
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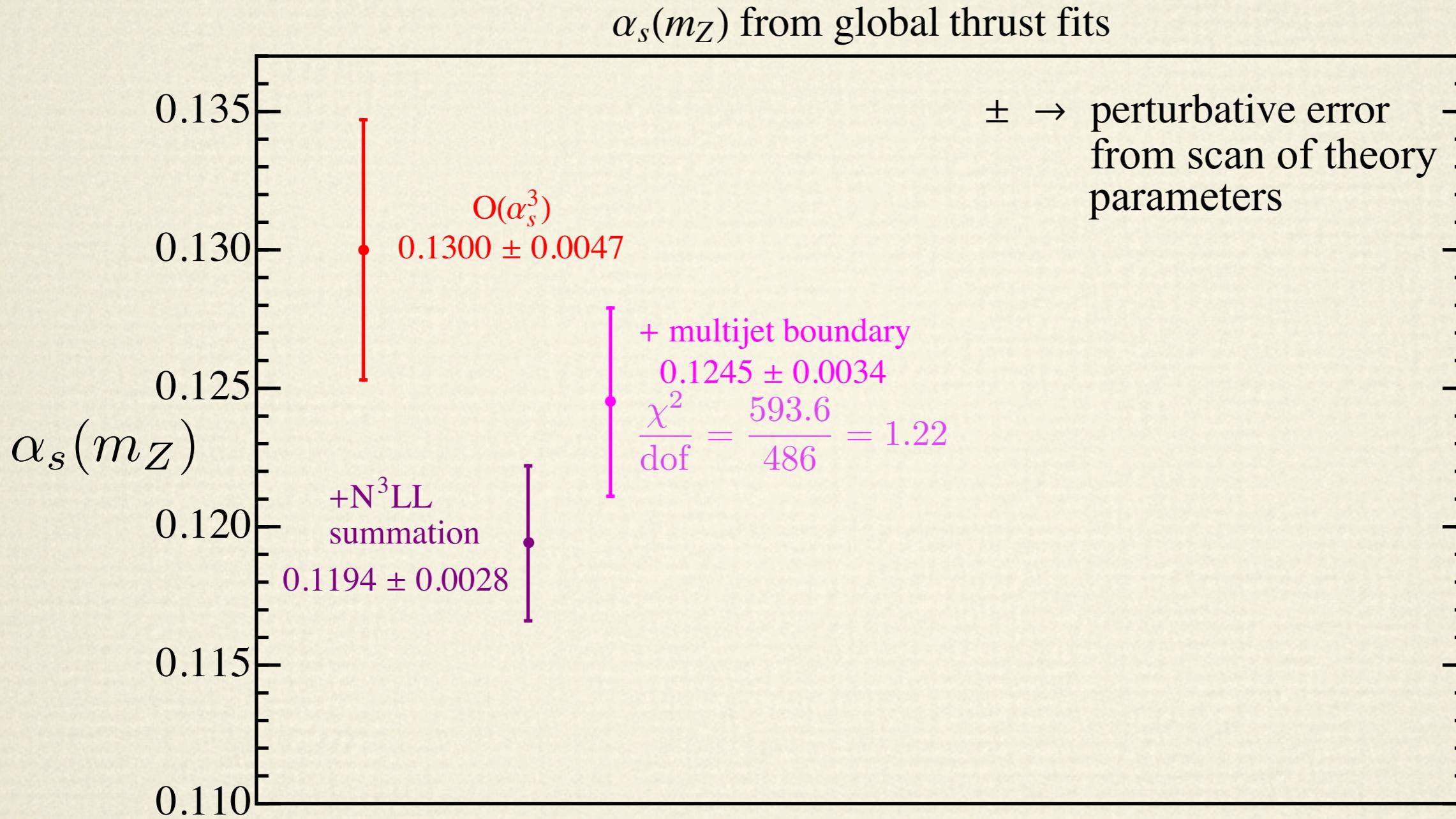


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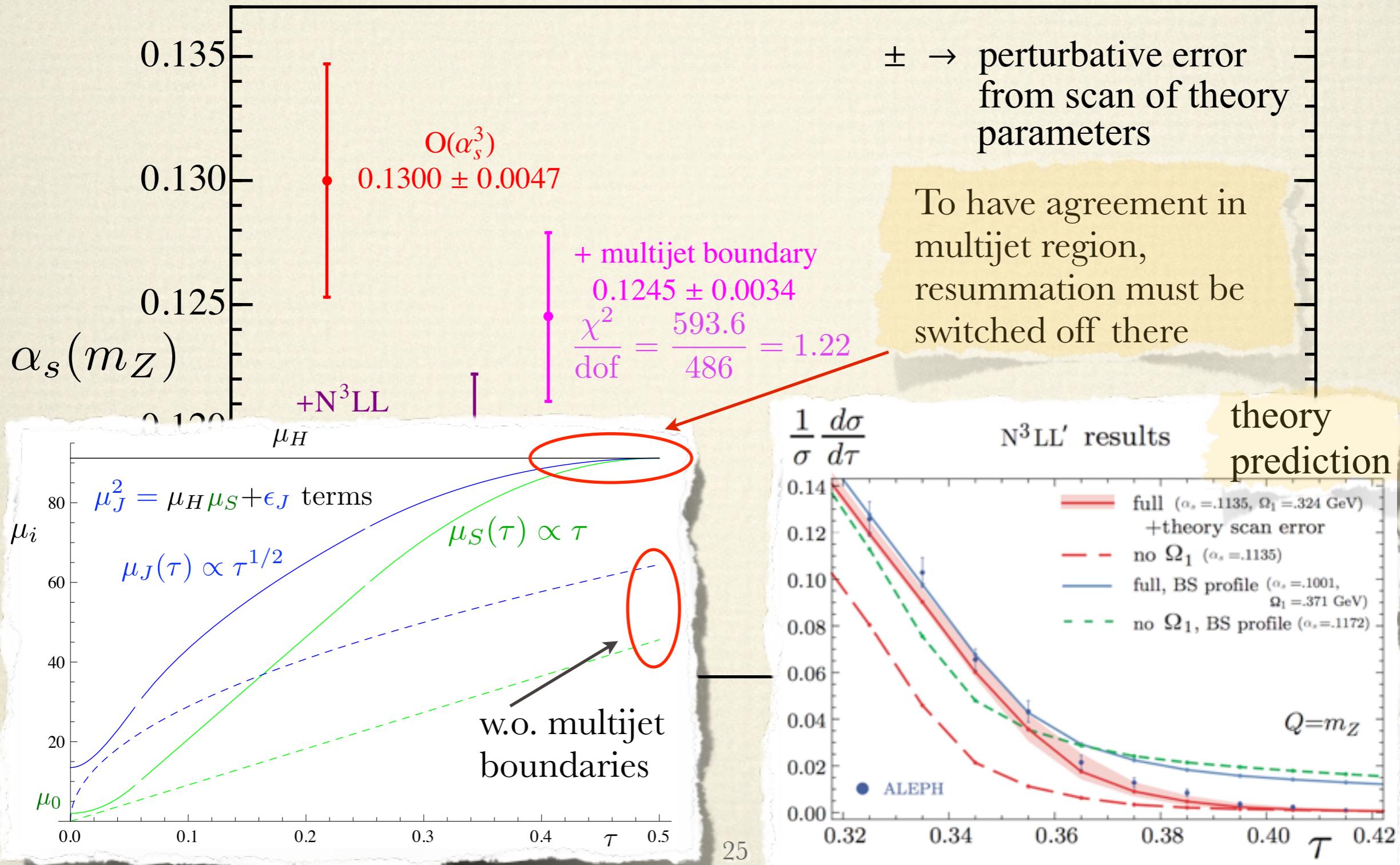


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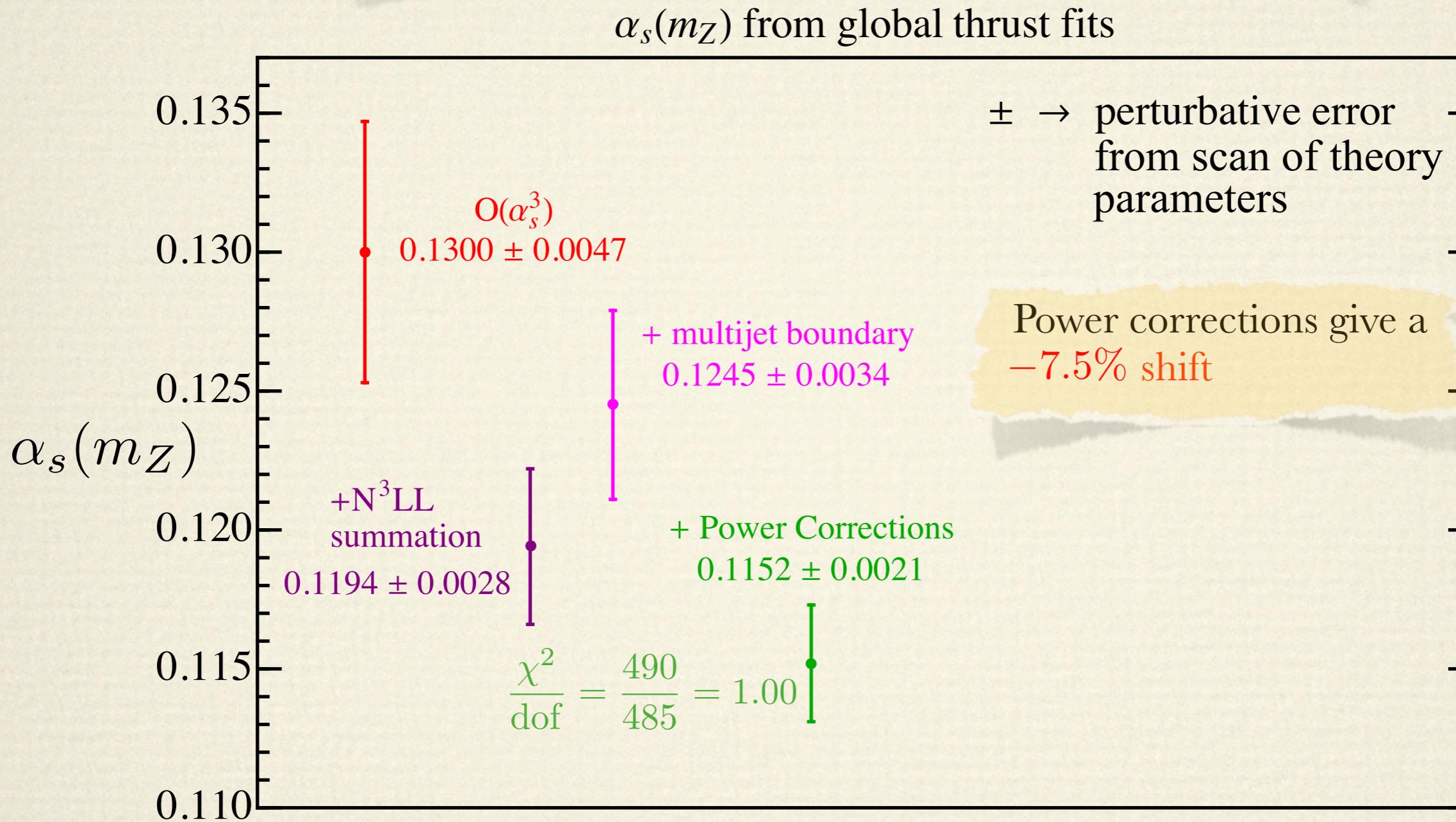


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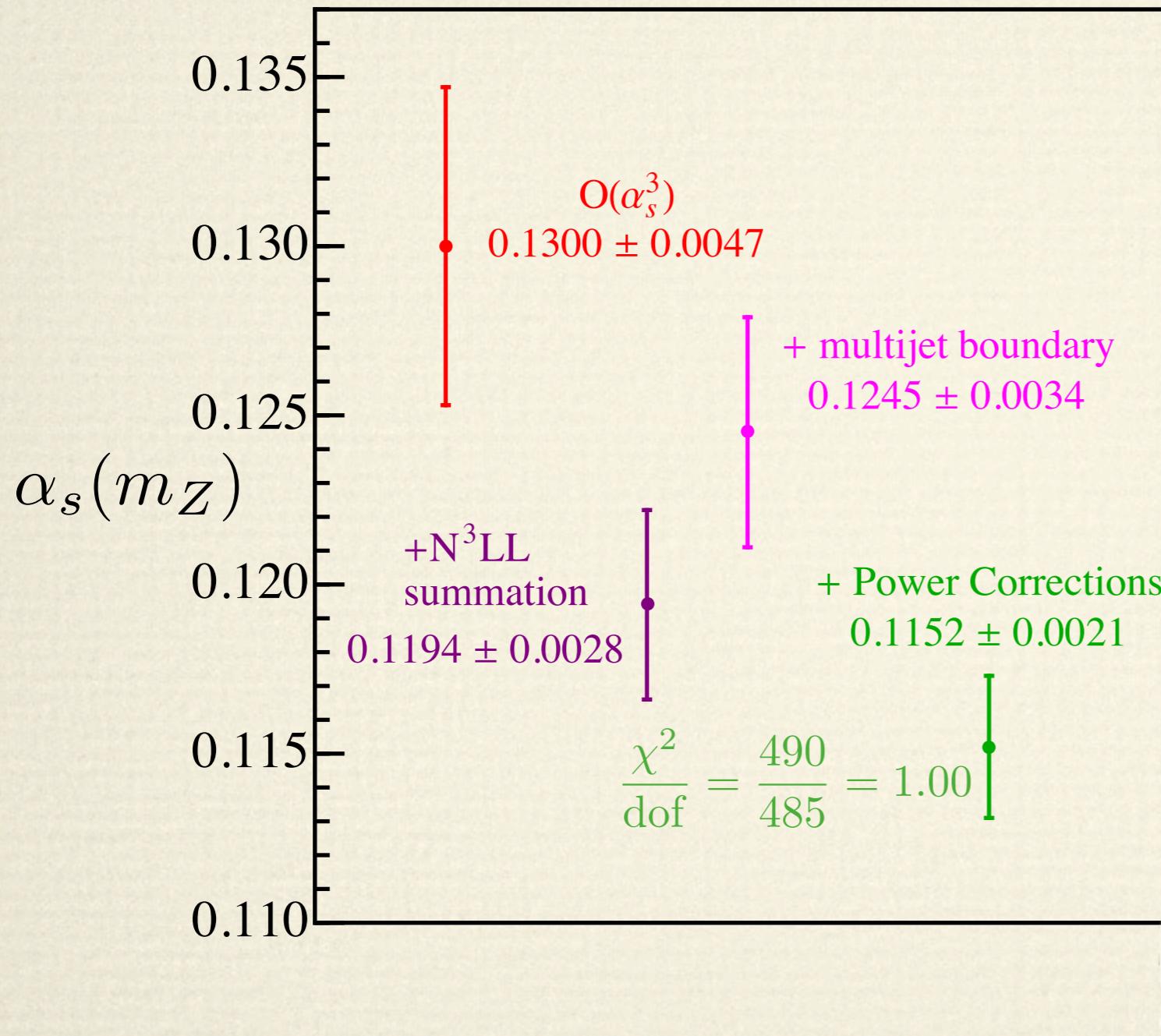


Global tail fit for $\alpha_s(m_Z)$ and Ω_1



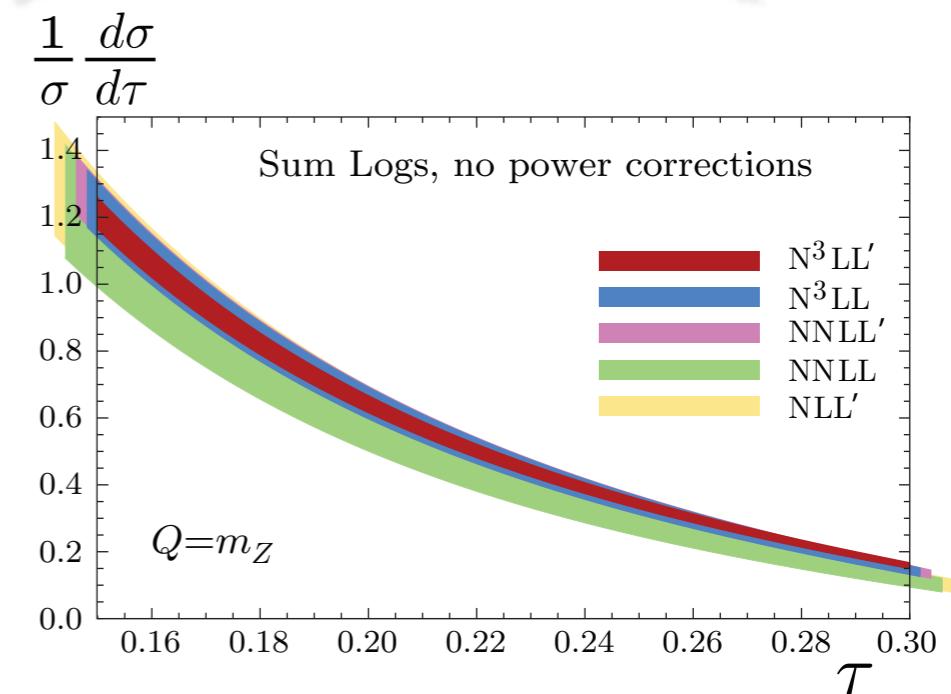
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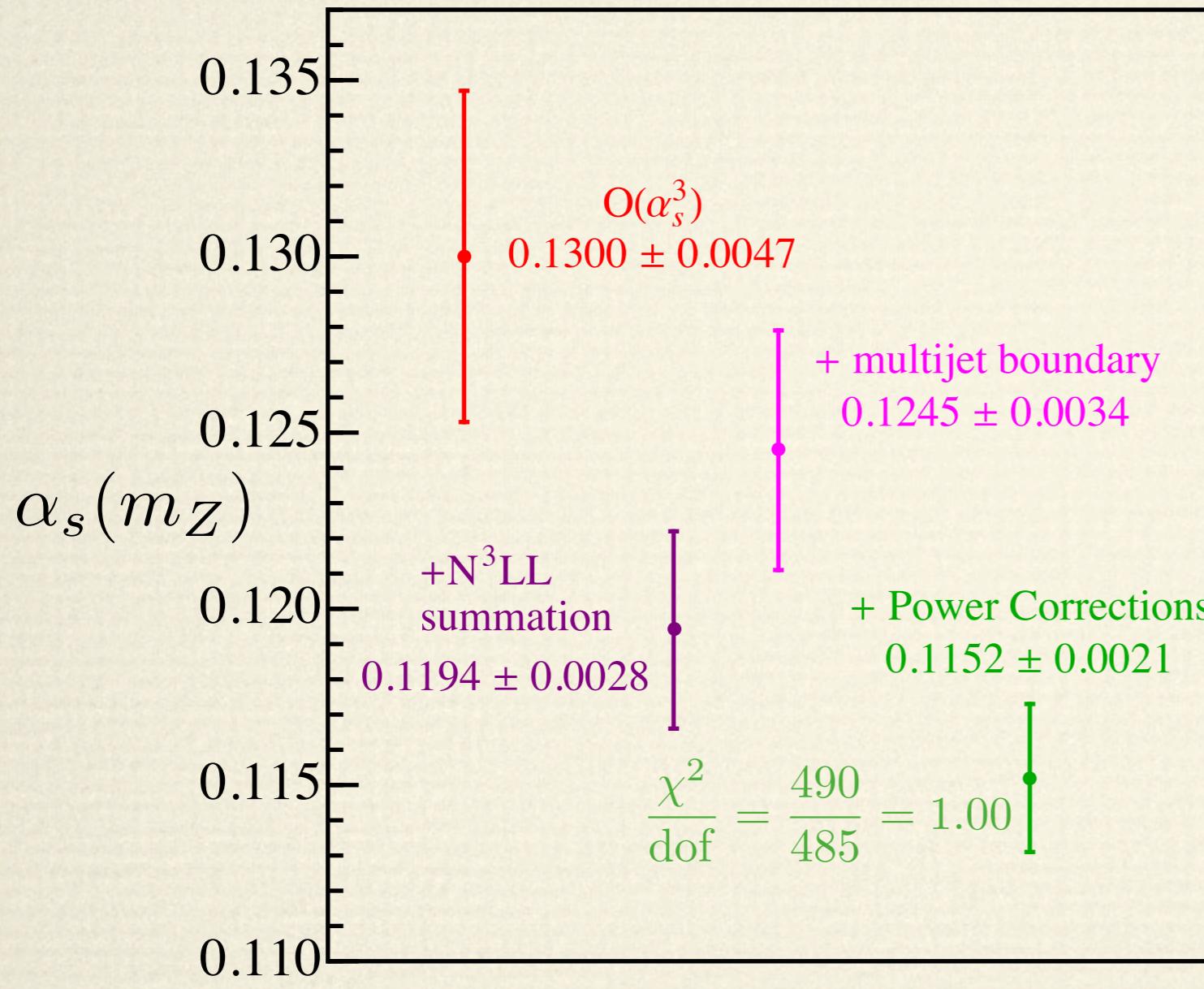
$\pm \rightarrow$ perturbative error
from scan of theory
parameters

Power corrections give a
-7.5% shift



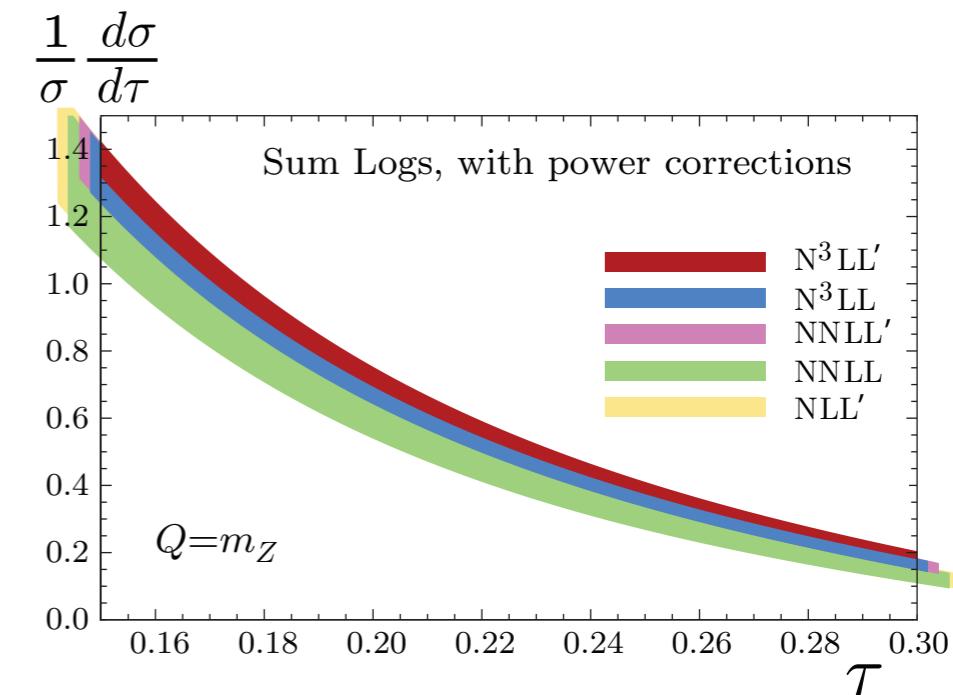
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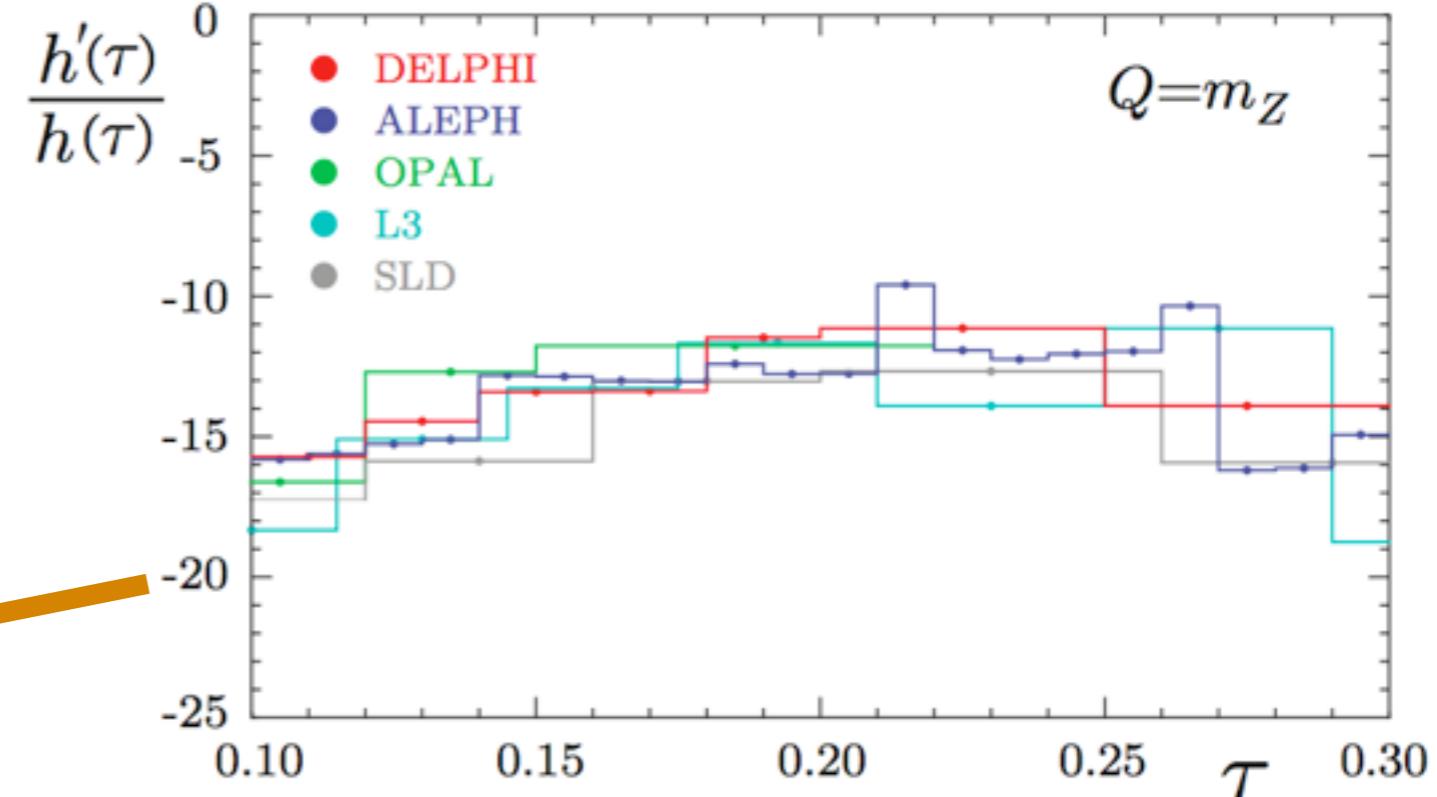
Leading power correction estimation

$$\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h \left(\tau - \frac{2\Lambda}{Q} \right)$$

Assuming $h \sim \alpha_s$

$$\frac{\delta \alpha_s}{\alpha_s} = \frac{2\Lambda}{Q} \frac{h'(\tau)}{h(\tau)}$$

$$\frac{h'(\tau)}{h(\tau)} = -14 \pm 4$$

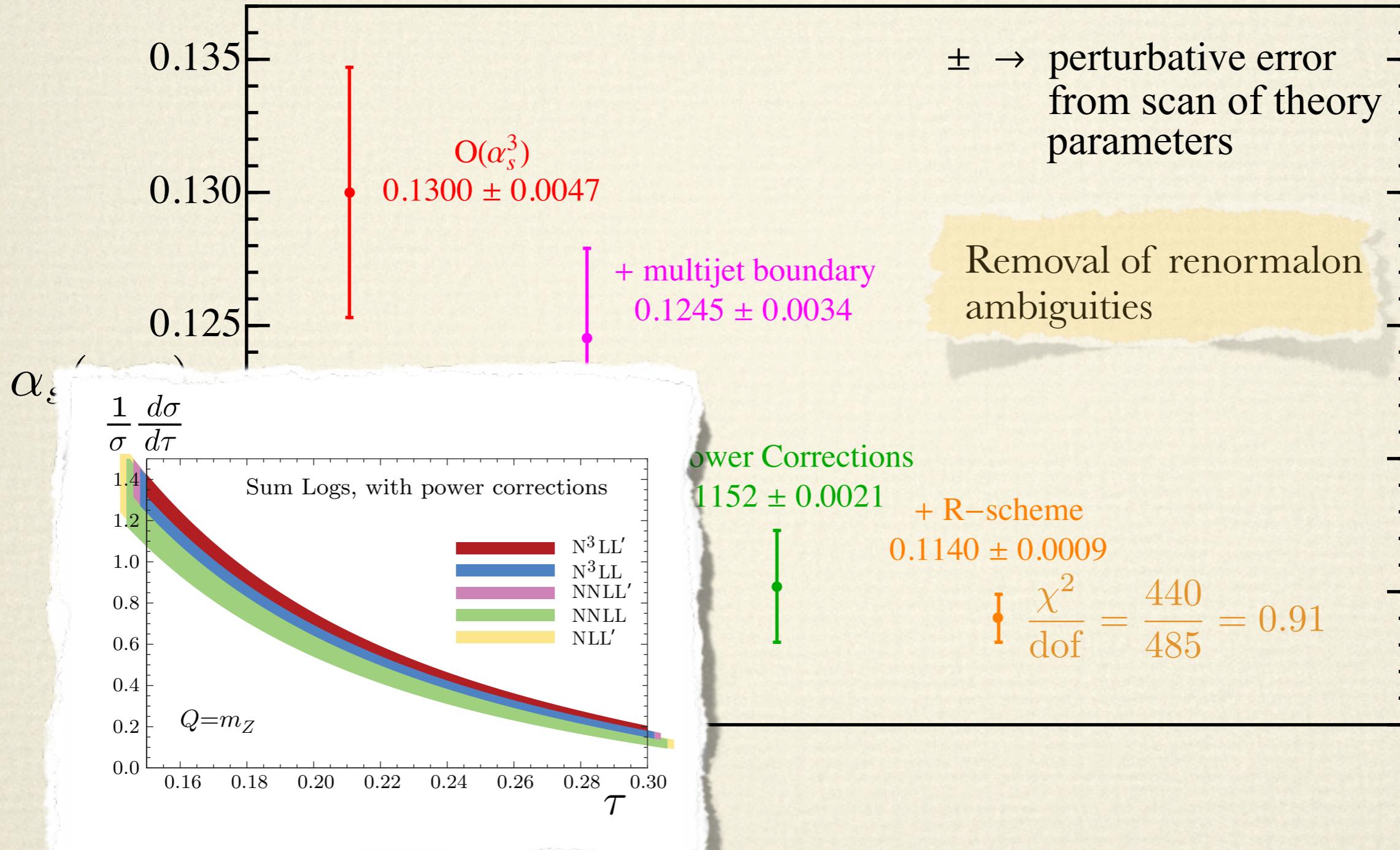


Assuming $\Lambda = 0.3 \text{ GeV}$

$$\frac{\delta \alpha_s}{\alpha_s} = -(9 \pm 3)\%$$

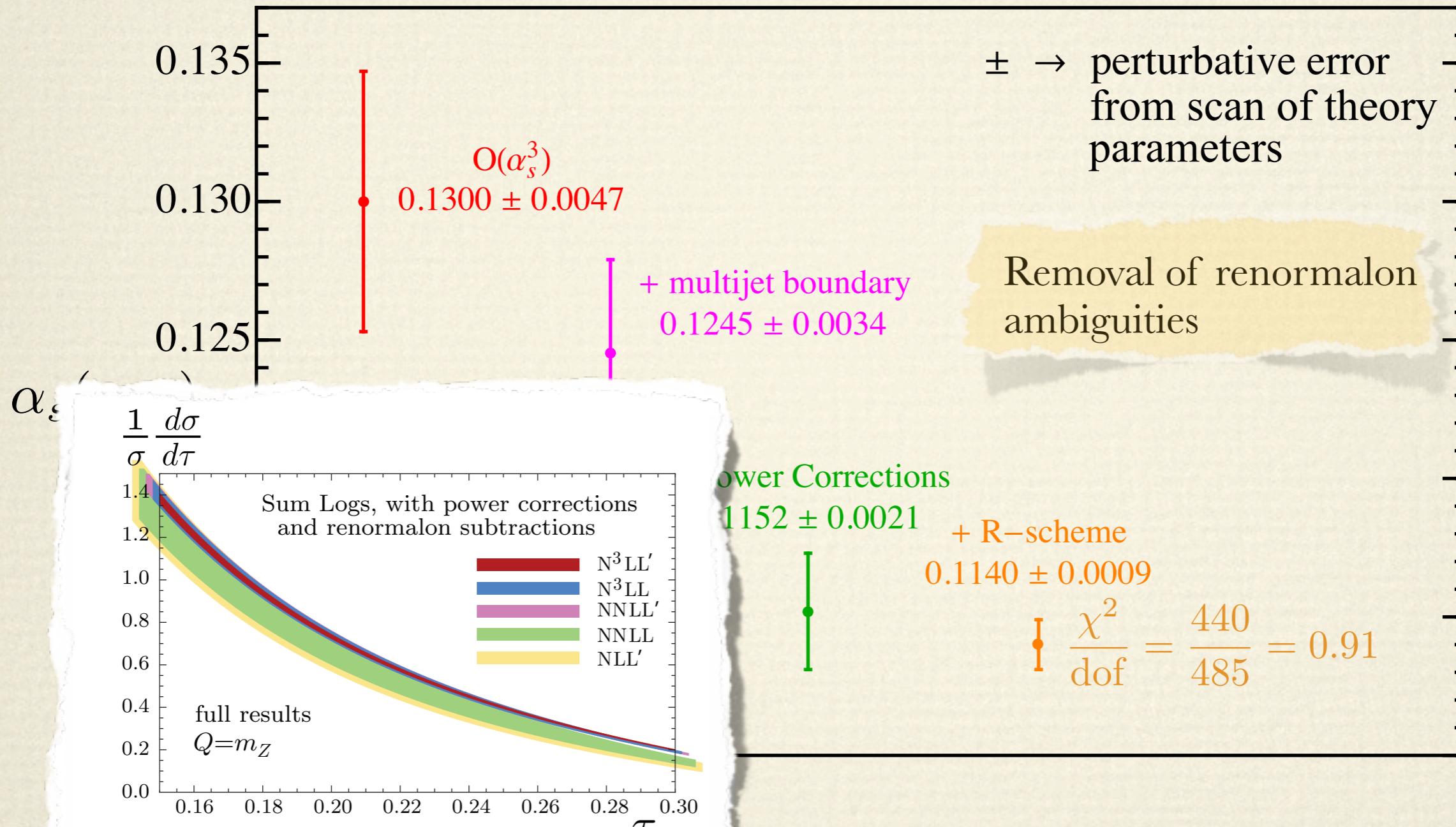
Global tail fit for $\alpha_s(m_Z)$ and Ω_1

$\alpha_s(m_Z)$ from global thrust fits

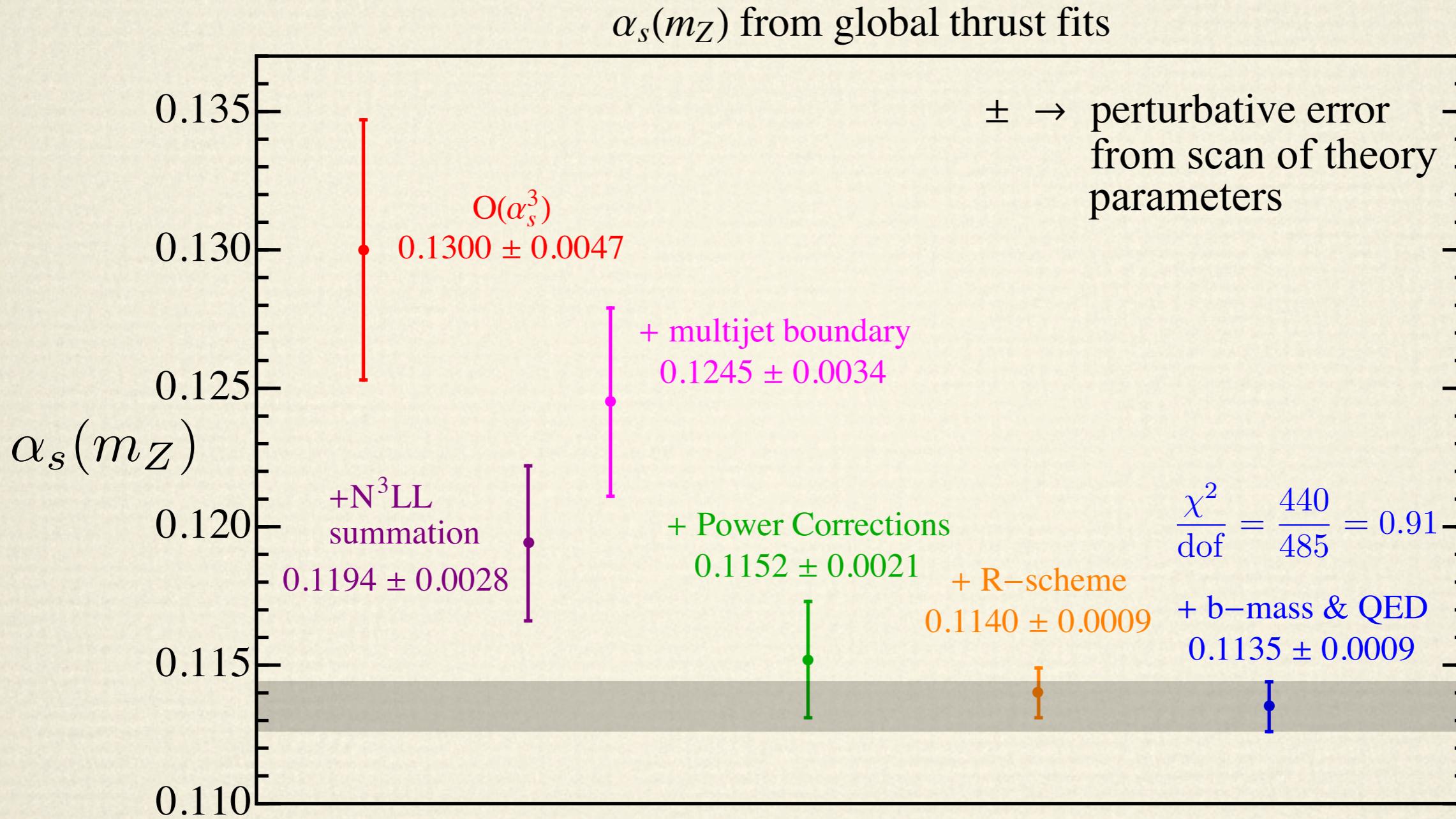


Global tail fit for $\alpha_s(m_Z)$ and Ω_1

$\alpha_s(m_Z)$ from global thrust fits



Global tail fit for $\alpha_s(m_Z)$ and Ω_1



Include b-mass effects in Factorization Thm: ($\sim 2\%$ effect)

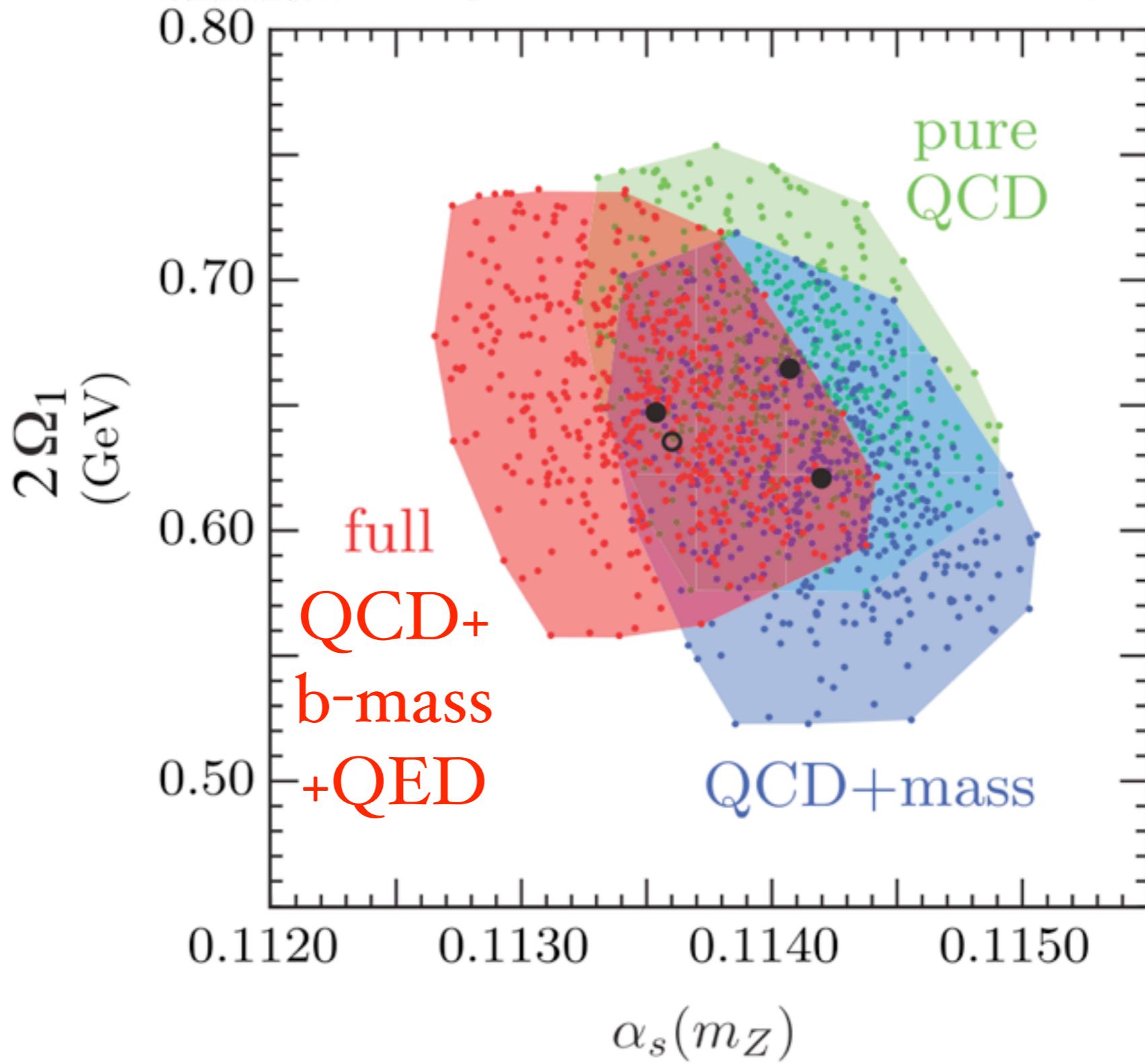
$$\frac{\Delta d\hat{\sigma}^b}{d\tau} = \frac{d\hat{\sigma}_{\text{massive}}^{NNLL}}{d\tau} - \frac{d\hat{\sigma}_{\text{massless}}^{NNLL}}{d\tau}$$

- at this order it effects only the jet function and τ limits
- use SCET massive fact. thm
- charm quarks are much smaller effect

Fleming, Hoang, Mantry, Stewart

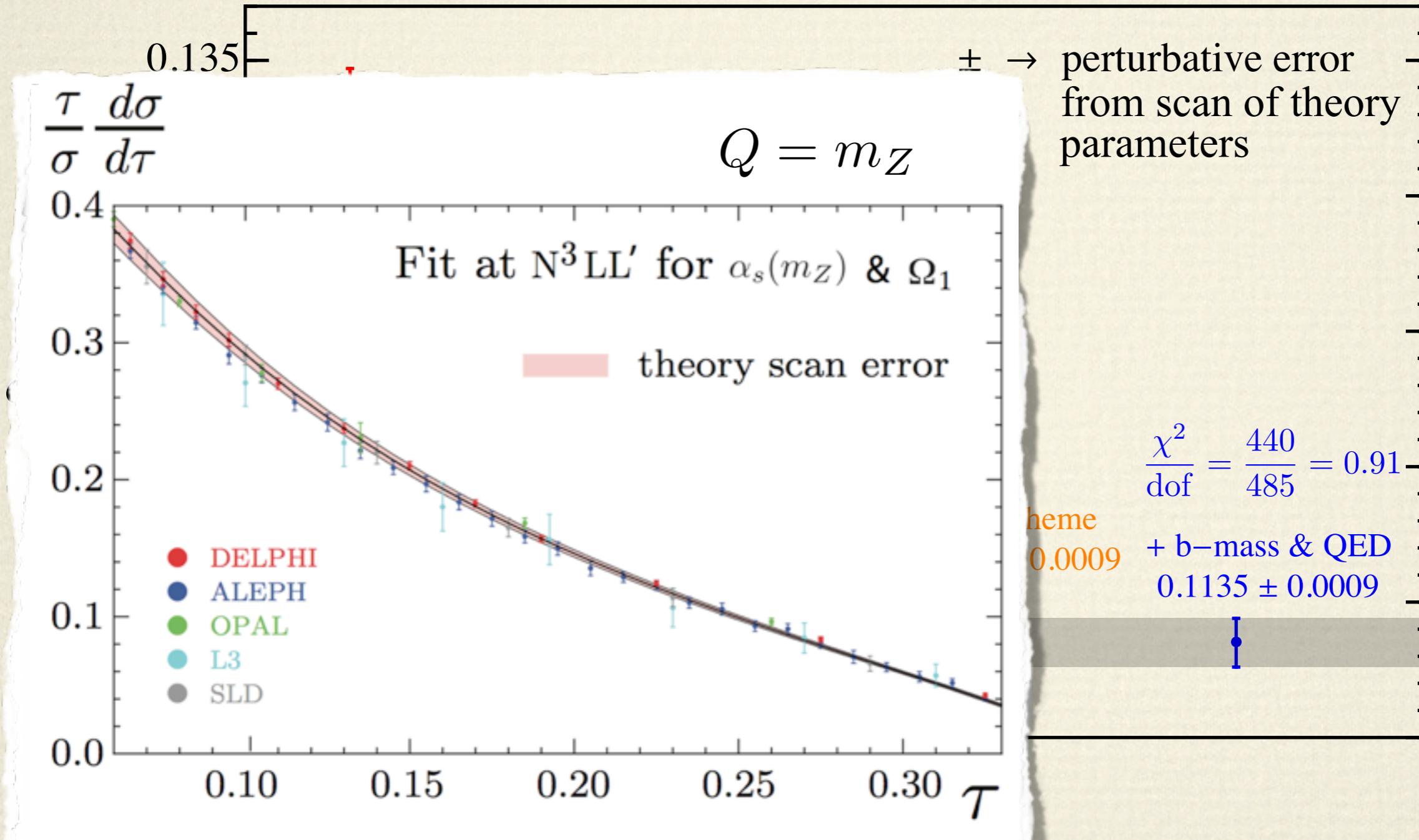
Include QED effects in Factorization Thm: ($\sim 2\%$ effect)

- count $\alpha \sim \alpha_s^2$, include only final state radiation
- include $\mathcal{O}(\alpha_s^2 \alpha)$ corrections to QCD β -function
- include one-loop QED corrections to H_Q, J_τ, S_τ

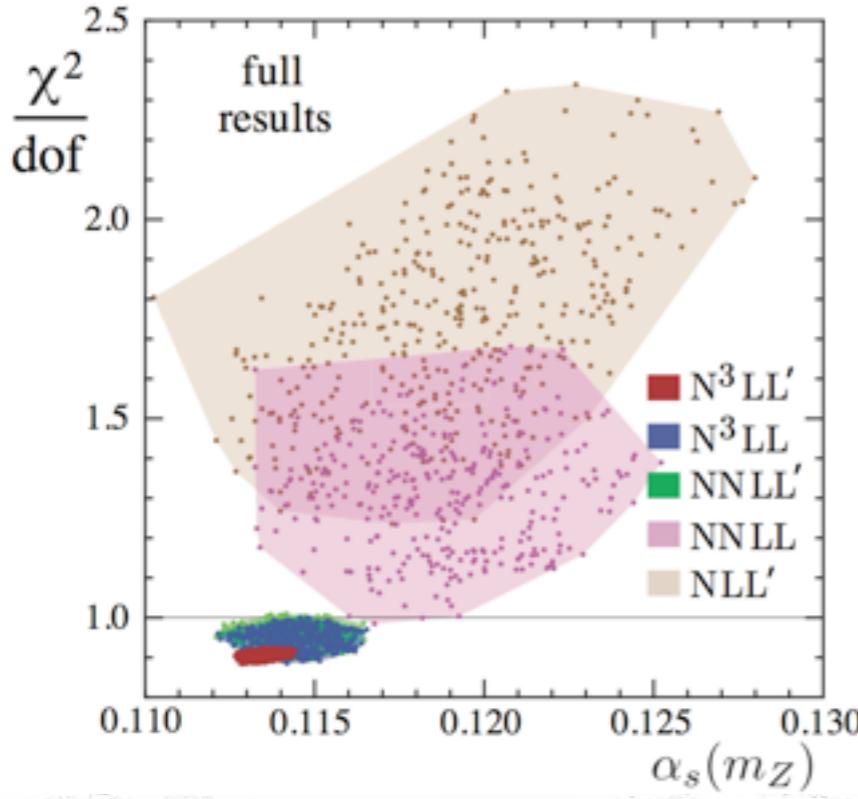
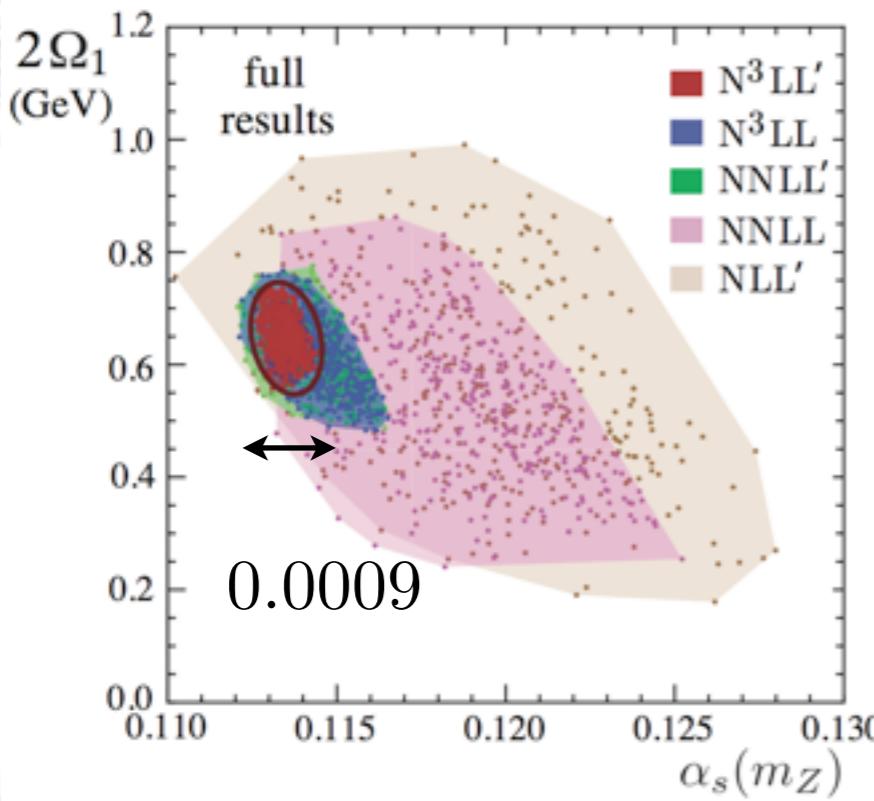


Global tail fit for $\alpha_s(m_Z)$ and Ω_1

$\alpha_s(m_Z)$ from global thrust fits



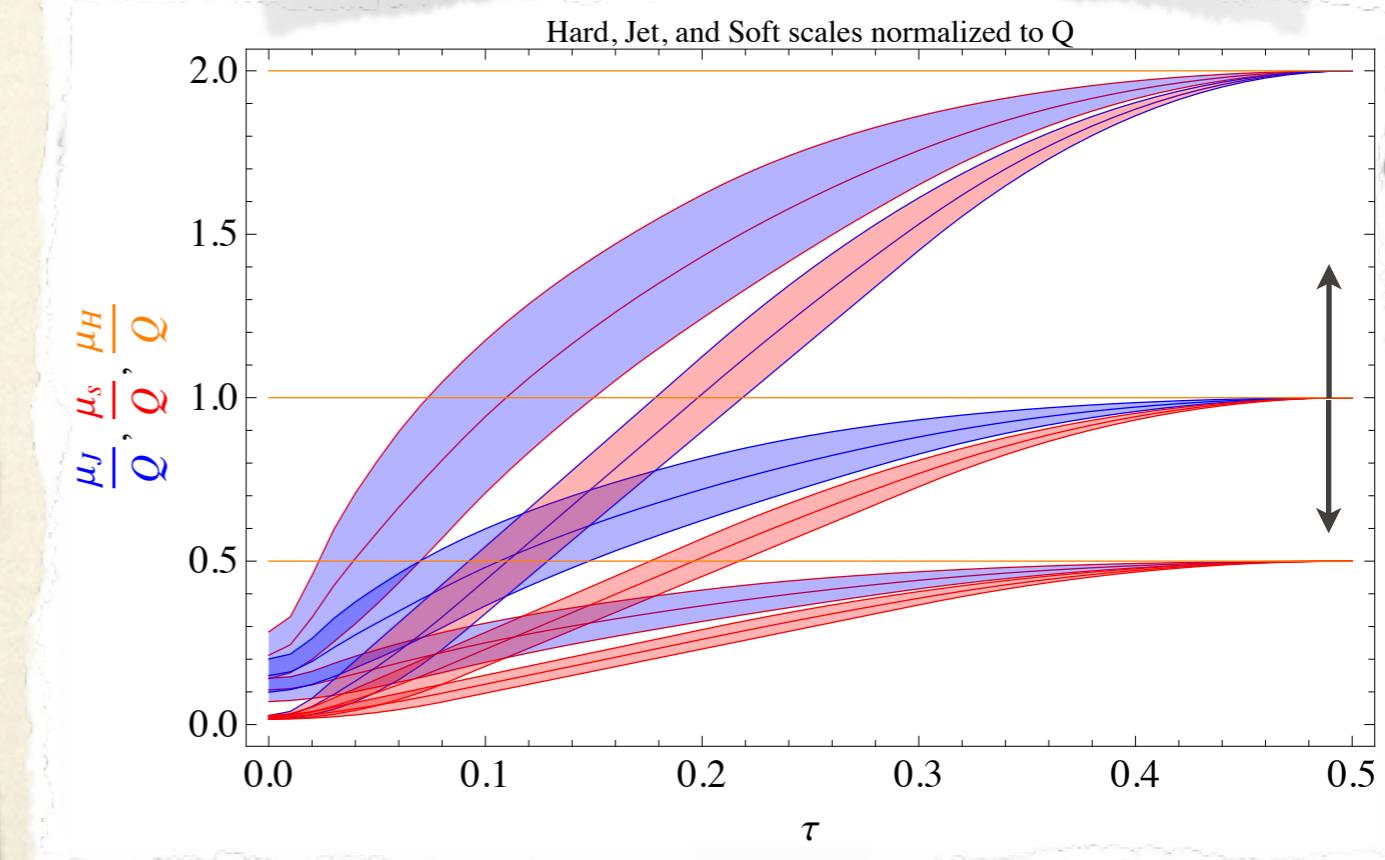
Convergence of results



Ω_1 determined to 16% accuracy

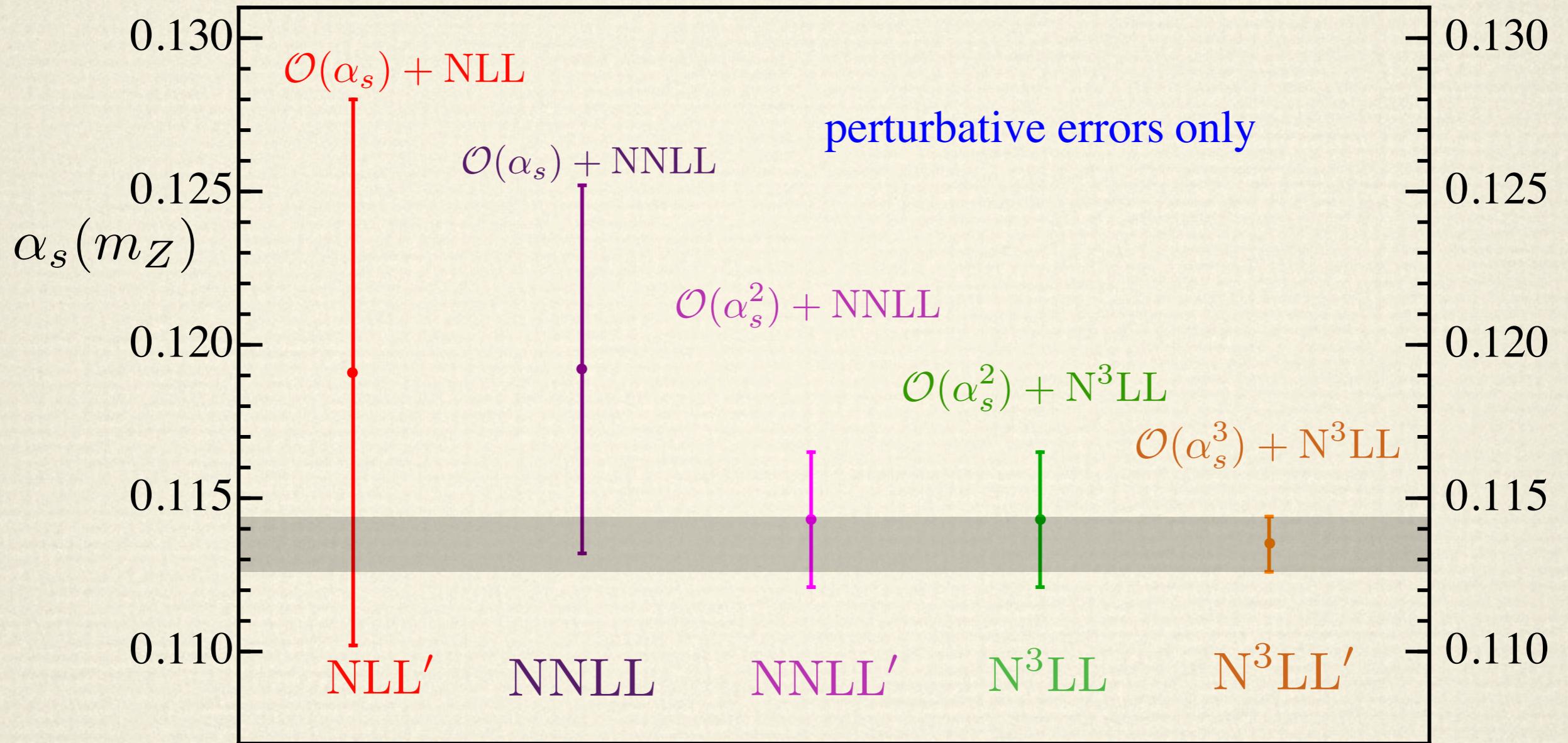
500-points random scan per order

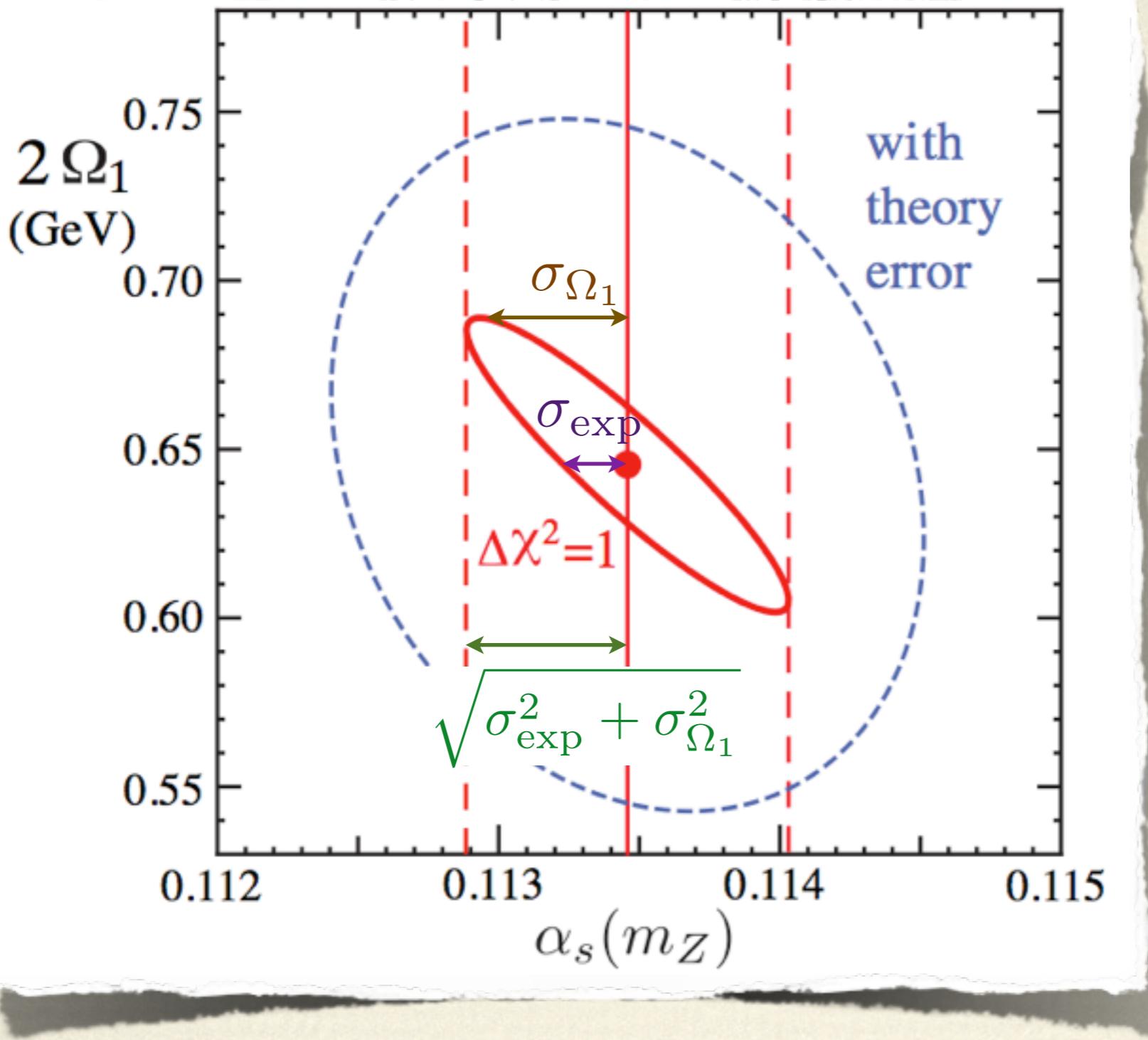
Largest contribution to perturbative uncertainty comes from variation of profile parameters



Convergence of results

$\alpha_s(m_Z)$ from global thrust fits





$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

$$\Omega_1 = 0.323 \pm 0.009_{\text{exp}} \pm 0.013_{\Omega_2} \pm 0.020_{\alpha_s(m_Z)} \pm 0.045_{\text{pert}} \text{ GeV}$$

$$\frac{\chi^2}{\text{dof}} = \frac{440}{485} = 0.91$$

“Standard” dataset

$$Q \geq 35 \text{ GeV}$$

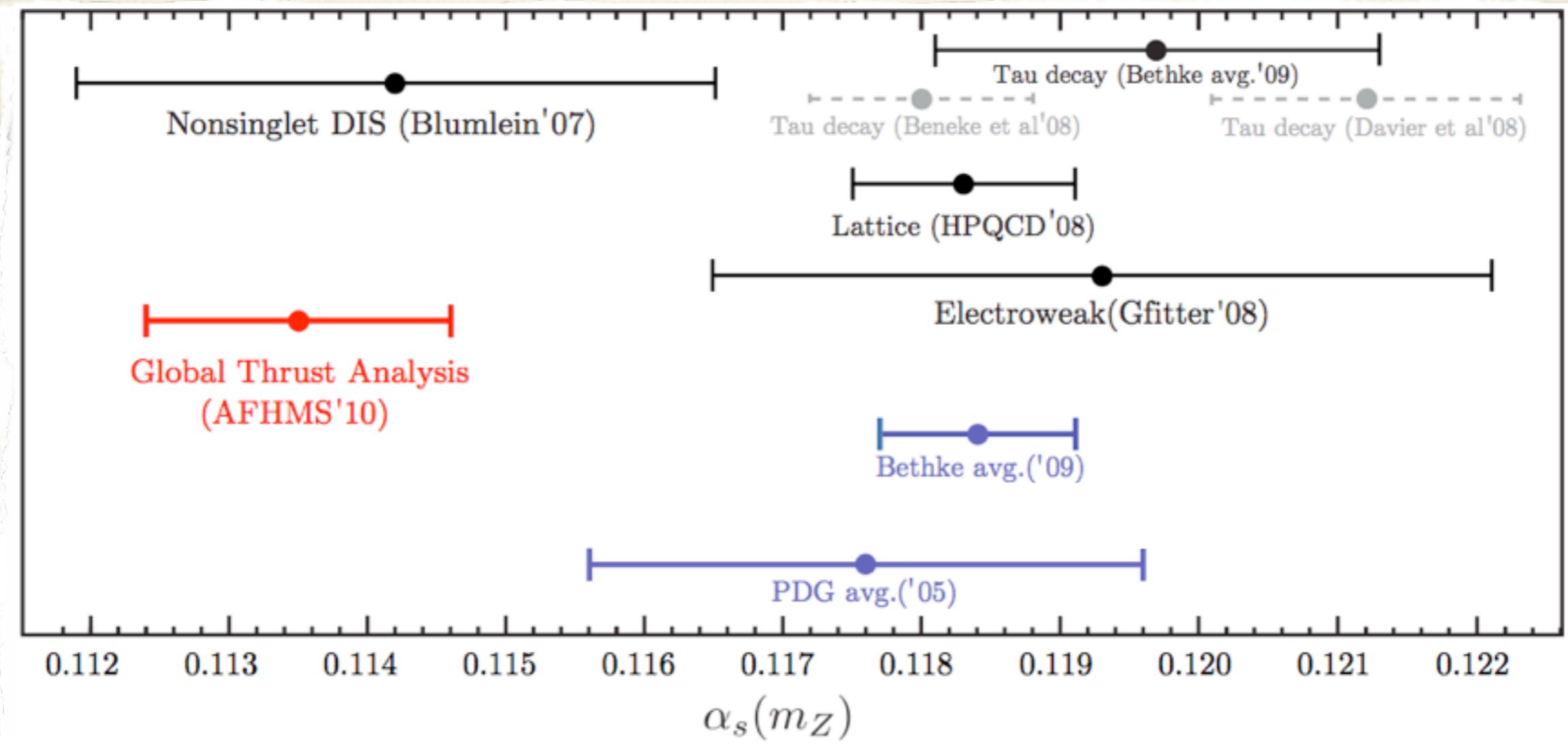
$$\frac{6 \text{ GeV}}{Q} \leq \tau \leq 0.33$$

487 bins

Correlations treated with
the minimal overlap
model

Final thrust results

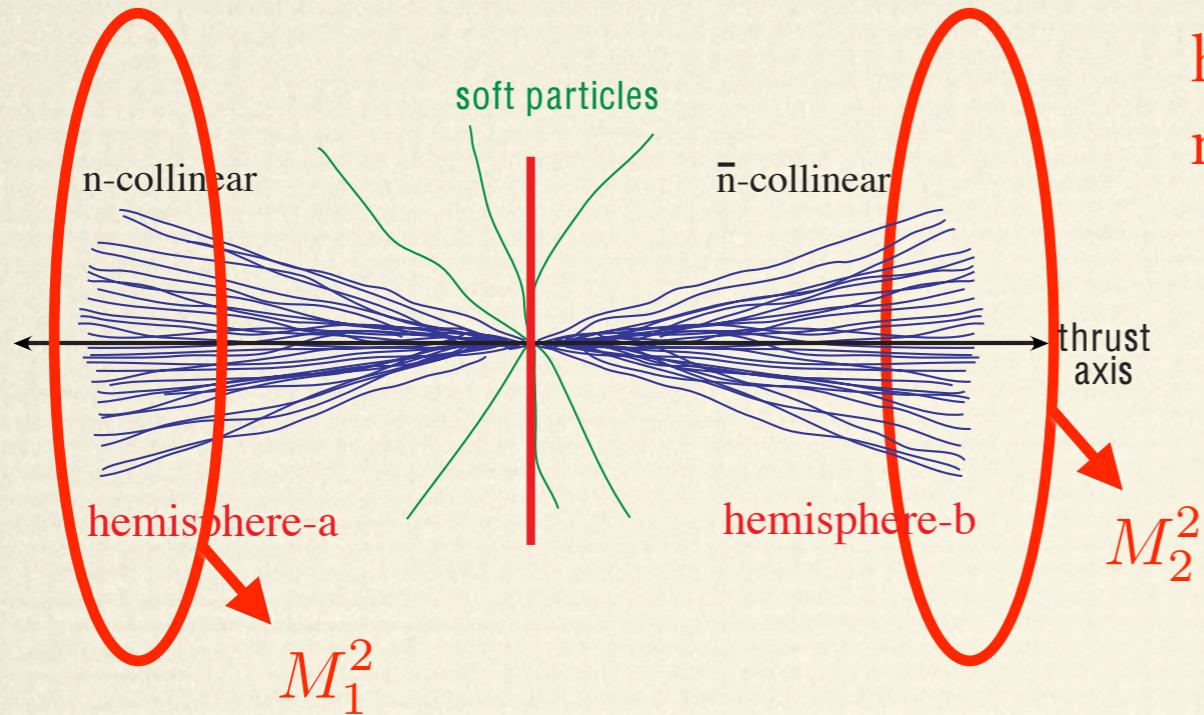
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Comparison with Heavy Jet Mass

Comparison with Heavy Jet Mass

AHMS&Schwartz
work in progress



hemisphere invariant
masses

$$M_i^2 = \left(\sum_{a \in i} p_a^\mu \right)^2$$

$$\rho = \max \left\{ \frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2} \right\}$$

Only ALEPH data $N^3LL + \mathcal{O}(\alpha_s^3)$

$$\alpha_s^\tau(m_Z) = 0.1175 \pm 0.0026$$

Becher&Schwartz '08

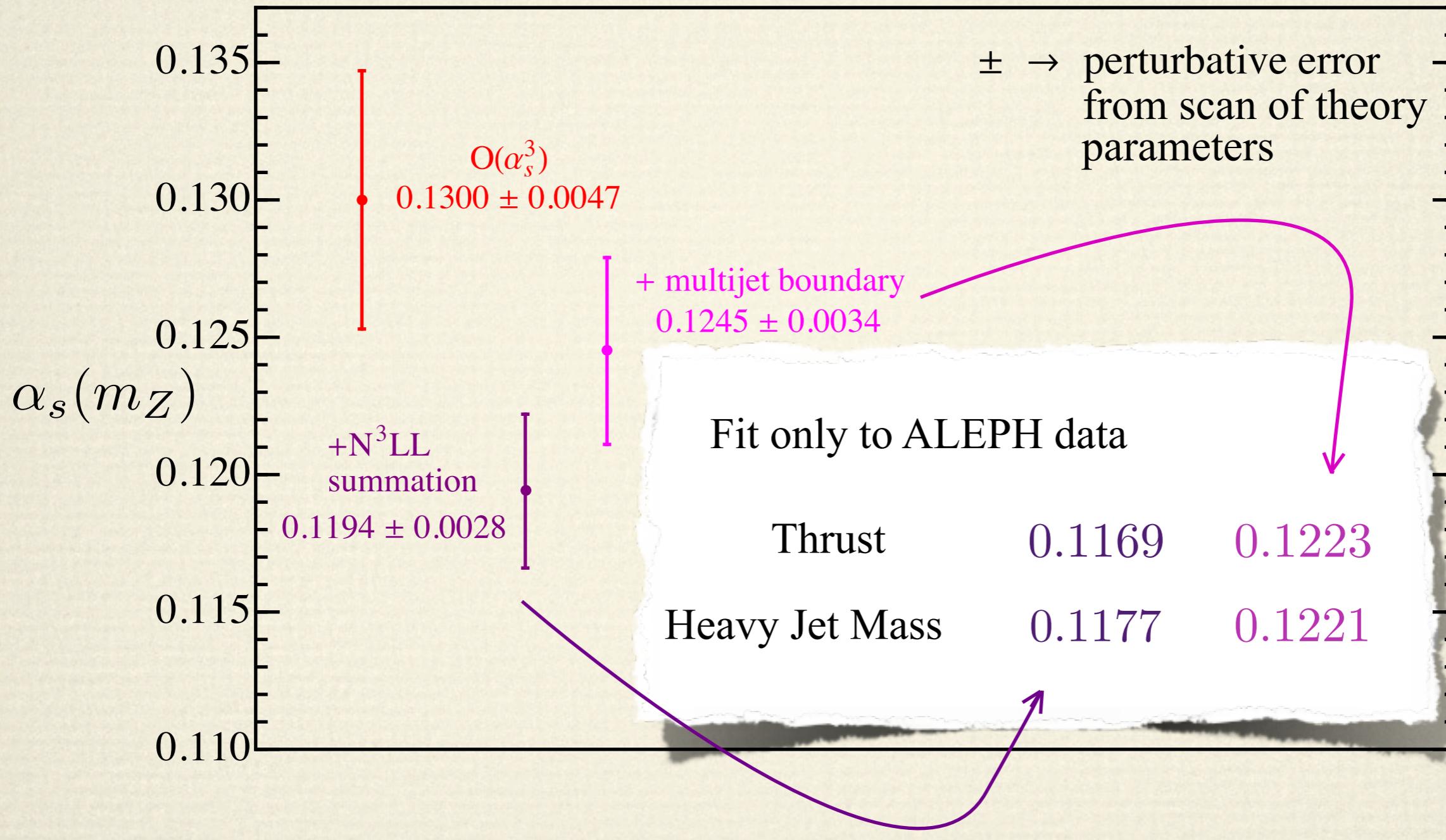
$$\alpha_s^\rho(m_Z) = 0.1220 \pm 0.0031$$

Chien&Schwartz '10

Comparison with Heavy Jet Mass

AHMS&Schwartz
work in progress

$\alpha_s(m_Z)$ from global thrust fits



Comparison with Heavy Jet Mass

AHMS&Schwartz
work in progress

There are two ways to calculate the theoretical value for a bin

Difference of Cumulants

$$\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))$$
$$\Sigma(\tau, \mu_i(\tau)) = \int_0^\tau dt \frac{d\sigma}{dt}(t, \mu_i(\tau))$$

Classical Resummation analyses

Becher&Schwartz, Chien&Schwartz

Integral of Differential Distribution

$$\int_{\tau_1}^{\tau_2} dt \frac{d\sigma}{dt}(t, \mu_i(t))$$

AFHMS, AHMS&Schwartz

Comparison with Heavy Jet Mass

AHMS&Schwartz
work in progress

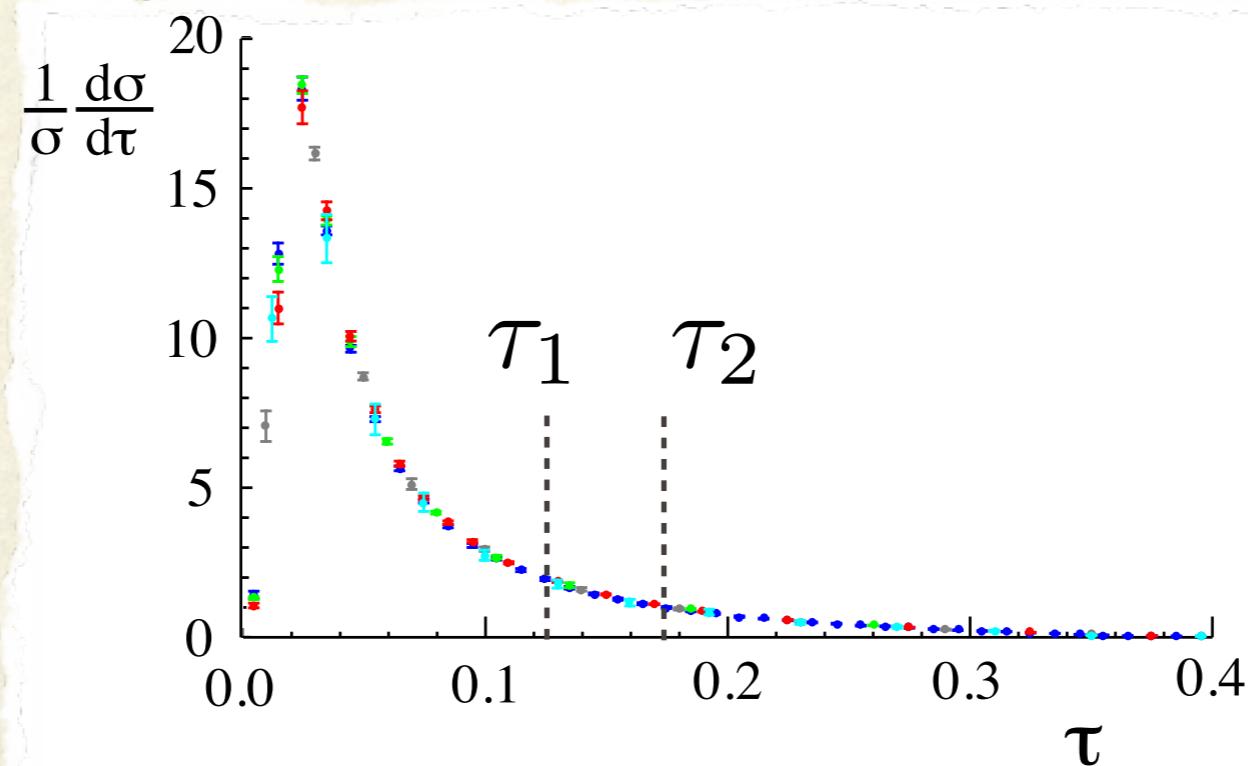
There are two ways to calculate the theoretical value for a bin

Difference of Cumulants

$$\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))$$

$$\begin{aligned} &= \int_{\tau_1}^{\tau_2} d\tau' \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau', \mu_i(\tau_2)) + \Sigma(\tau_1, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) \\ &\simeq \int_{\tau_1}^{\tau_2} d\tau' \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau', \mu_i(\tau')) + (\tau_2 - \tau_1) \frac{d\mu_i(\tau_1)}{d\tau} \frac{\partial}{\partial \mu_i} \int_0^{\tau_1} d\tau' \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau', \mu_i(\tau_1)) \end{aligned}$$

Integral of Differential Distribution



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Difference of Cumulants

Integral of Differential Distribution

$Q=91.2 \text{ GeV}$	No Power Corrections	
	resummation	multijet boundary
Cumulant	0.1217	0.1212
Integrated	0.1177	0.1221

HJM
 $N^3 LL + \mathcal{O}(\alpha_S^3)$

$$\alpha_s^\rho(m_Z) = 0.1220 \pm 0.0031$$

Chien&Schwartz '10

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Chien&Schwartz '10

- With power correction and gap, using $\Omega_1 = 0.332 \pm 0.045 \text{ GeV}$ we obtain

$$\alpha_s^\rho(m_Z) = 0.1133 \quad \alpha_s^\tau(m_Z) = 0.1140 \pm 0.0011$$

(no QED, no b-mass)

Conclusions

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets
- SCET has finally provided theorists with a means to catch up with the experimental precision of LEP. The result of our global thrust fit in the tail region is

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

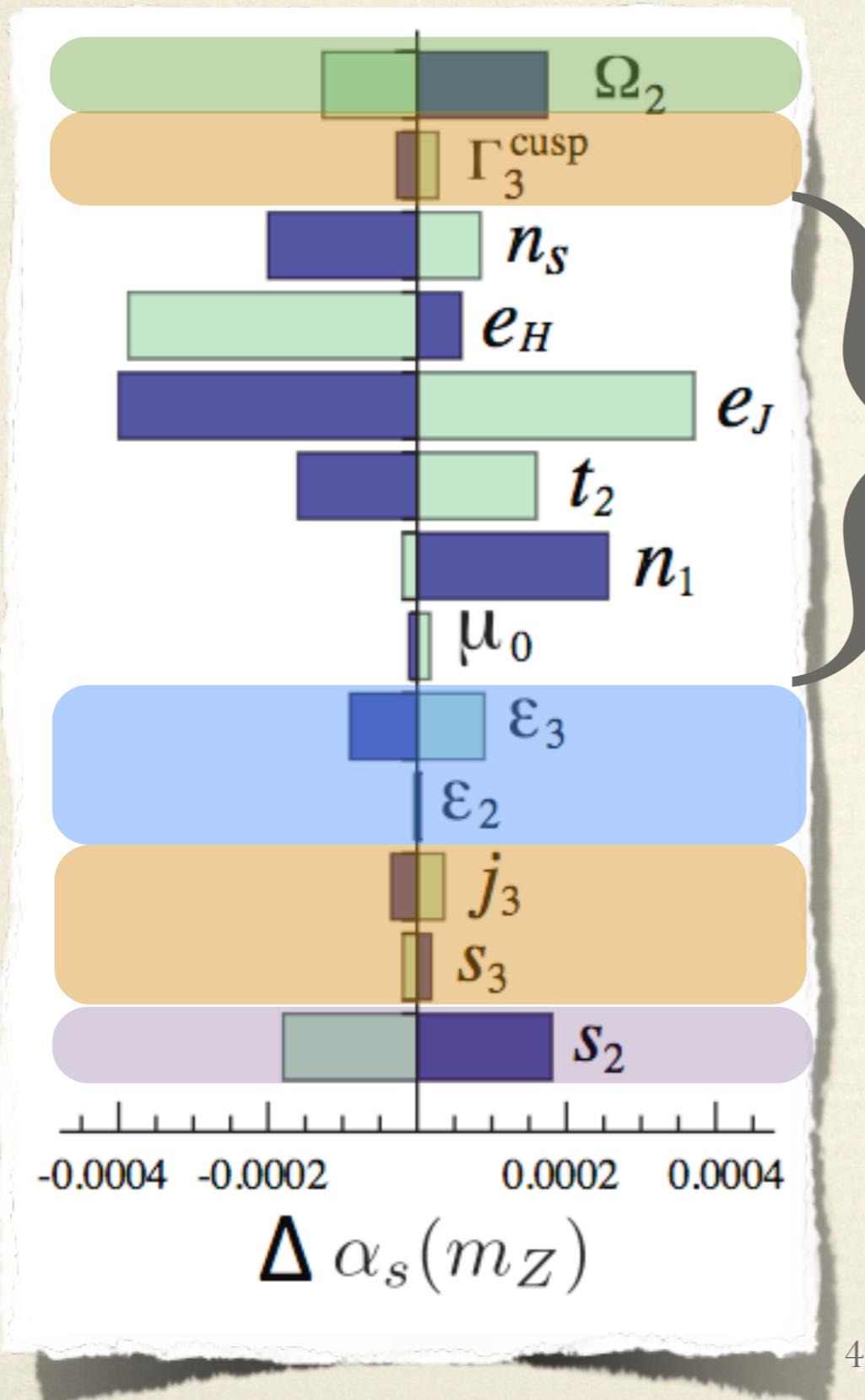
- Rigorous field theoretical treatment of nonperturbative effects

$$\Omega_1 = 0.323 \pm 0.009_{\text{exp}} \pm 0.013 \Omega_2 \pm 0.020 \alpha_s(m_Z) \pm 0.045_{\text{pert}} \text{ GeV}$$

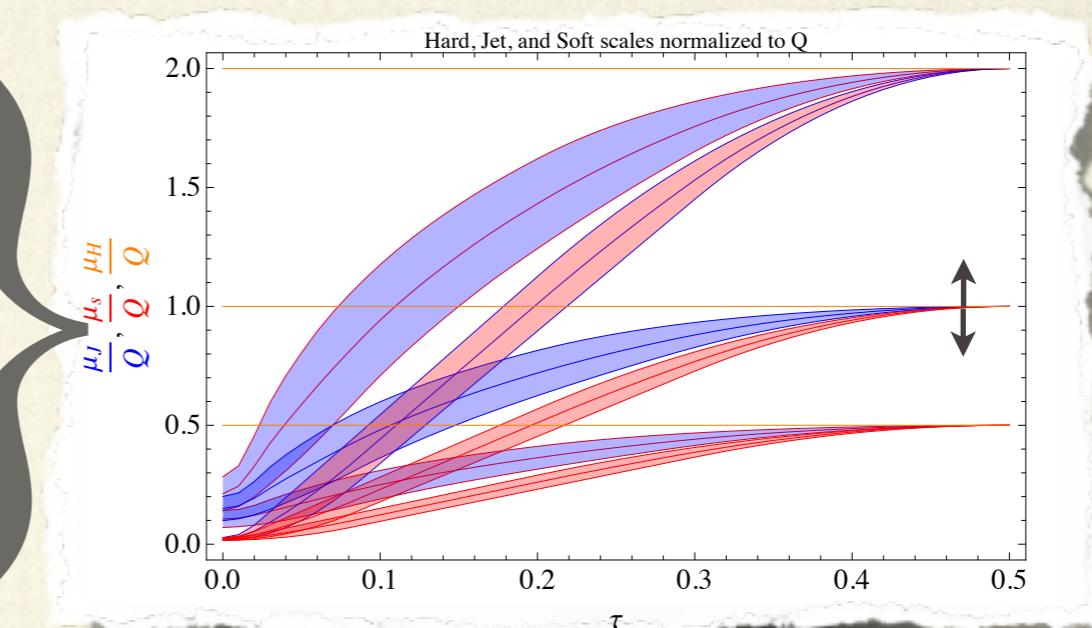
- Thrust and Heavy Jet Mass agree at perturbative level if bins are computed using the integral of the differential distribution instead of difference of cumulants

Backup slides

Uncertainty budget from scan



Uncertainty from higher moments



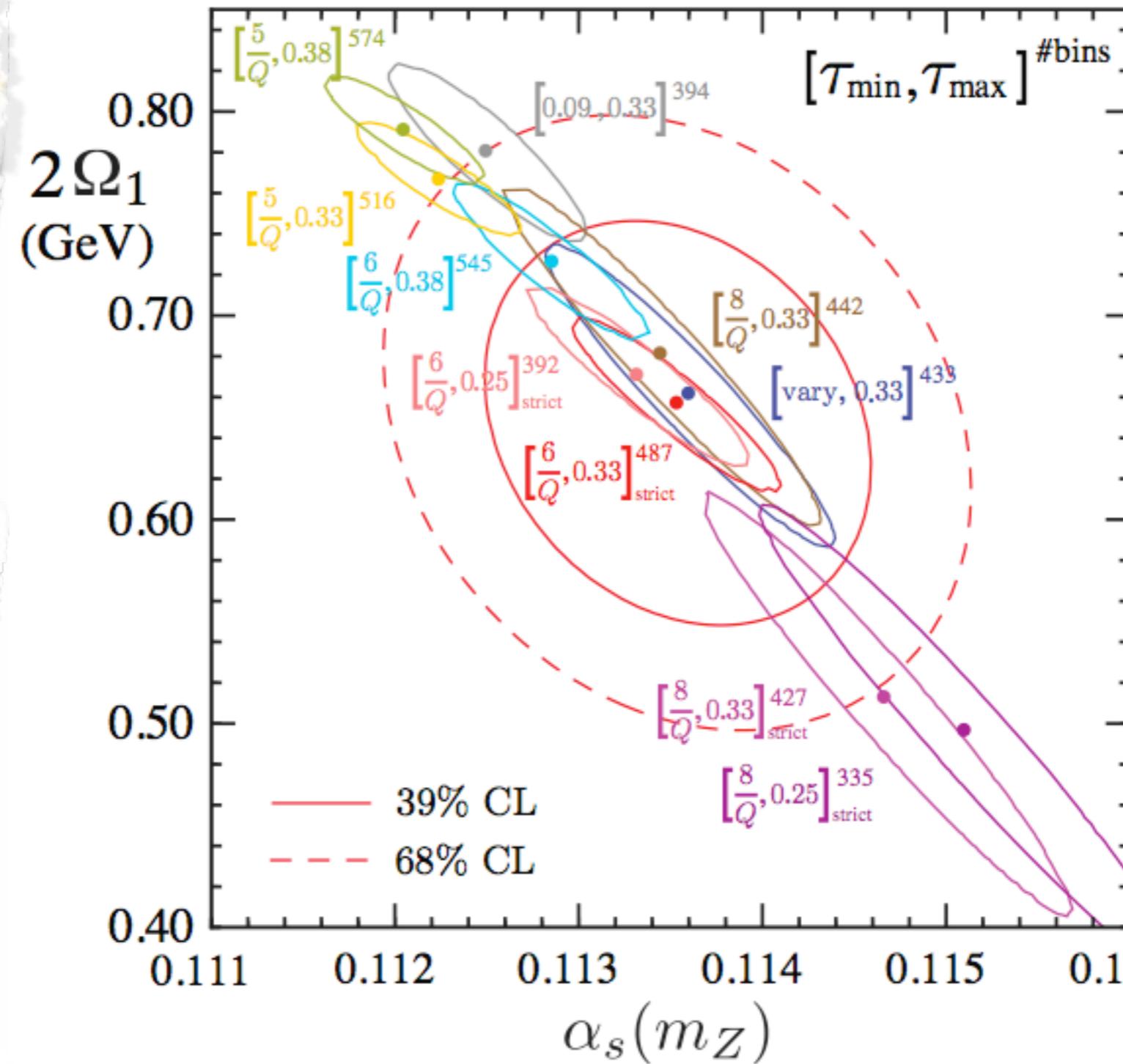
Statistical error from nonsingular extraction

$\mathcal{O}(\alpha_s^3)$ unknown. Estimated with Padè

$\mathcal{O}(\alpha_s^2)$ extracted numerically from EVENT2 with an accuracy of 6.4%

Different datasets

Ω_2 effect increases



Statistical errors increase

Cross section components

